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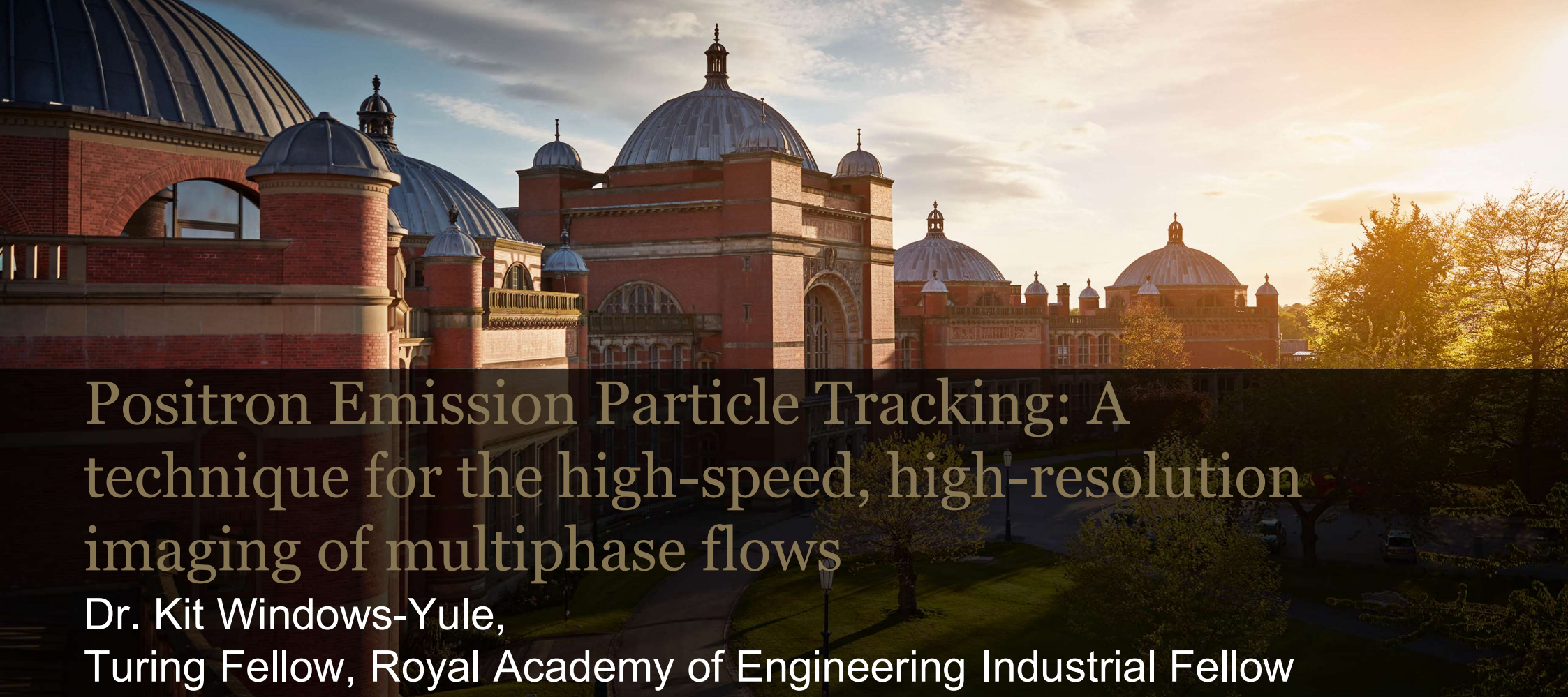


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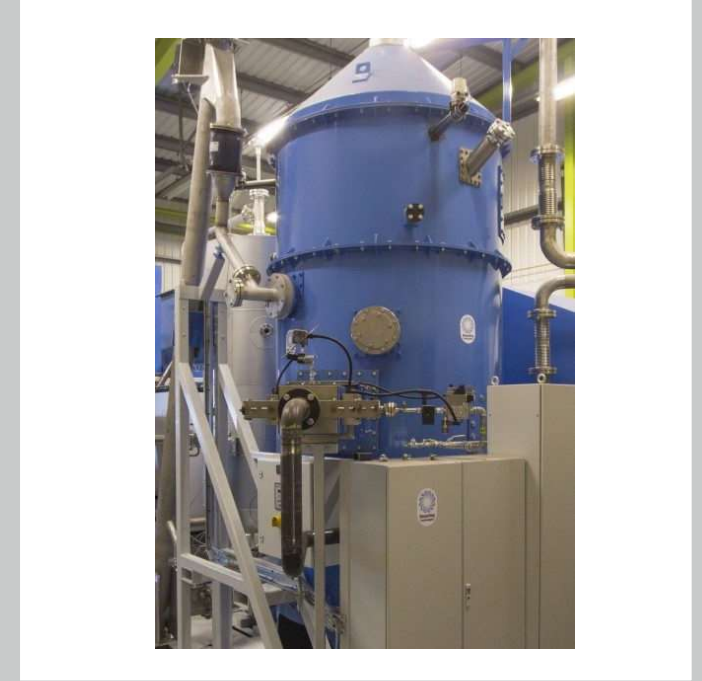
# Positron Emission Particle Tracking: A technique for the high-speed, high-resolution imaging of multiphase flows

Dr. Kit Windows-Yule,  
Turing Fellow, Royal Academy of Engineering Industrial Fellow



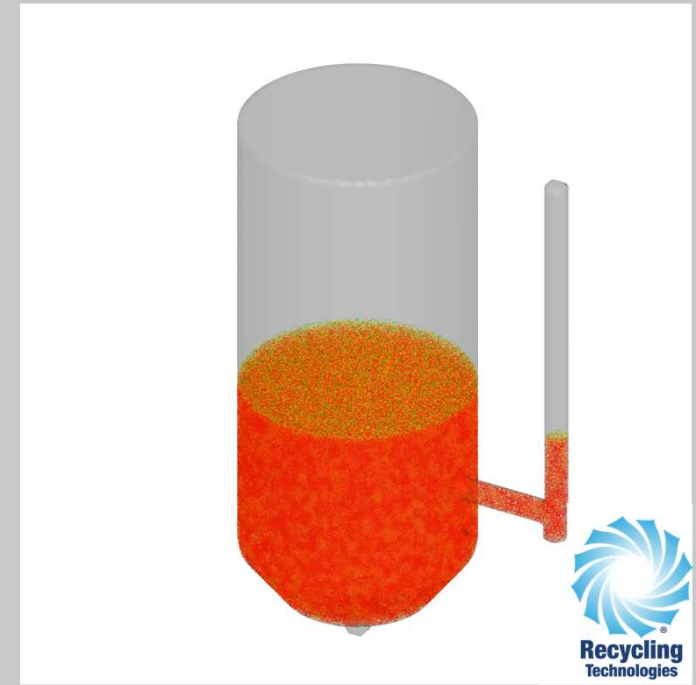
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# Overview & Motivation



## How do we better understand industrial systems?

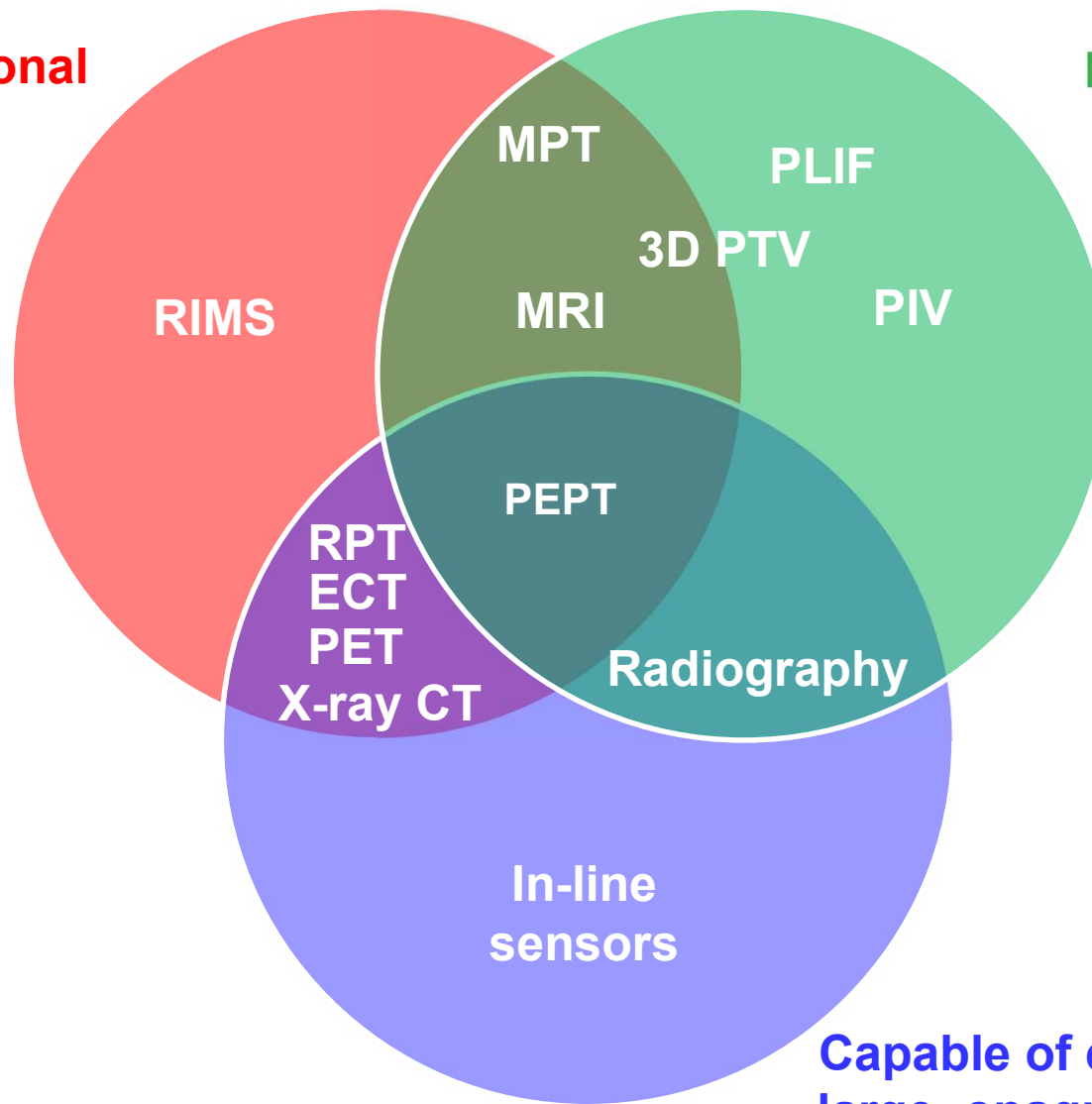
- Understanding the dynamics of multiphase flows is crucial to the optimisation of diverse process equipment
- ...but these large, metal systems are near-impossible to accurately image using conventional methods



## How do we better understand industrial systems?

- Numerical methods (DEM, CFD, MP-PIC...) can provide insight
- **But** without experimental validation, simulations may be **misleading**
- → We still need to find a way to experimentally investigate these large, opaque systems!

**Full, three-dimensional  
imaging**



**High temporal and  
spatial resolution**

**Capable of extracting data from  
large, opaque, metallic systems**

# Talk Overview



**I) An Introduction to PEPT**



**II) Case Study**



**III) The Synergy of PEPT and  
Numerical Simulation**



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# I. An Introduction to PEPT

# Positron Emission Particle Tracking (PEPT)

Uses highly-penetrating gamma radiation to **directly track** the three-dimensional motion of particulate, fluid and multiphase systems, with **high temporal and spatial resolution**.

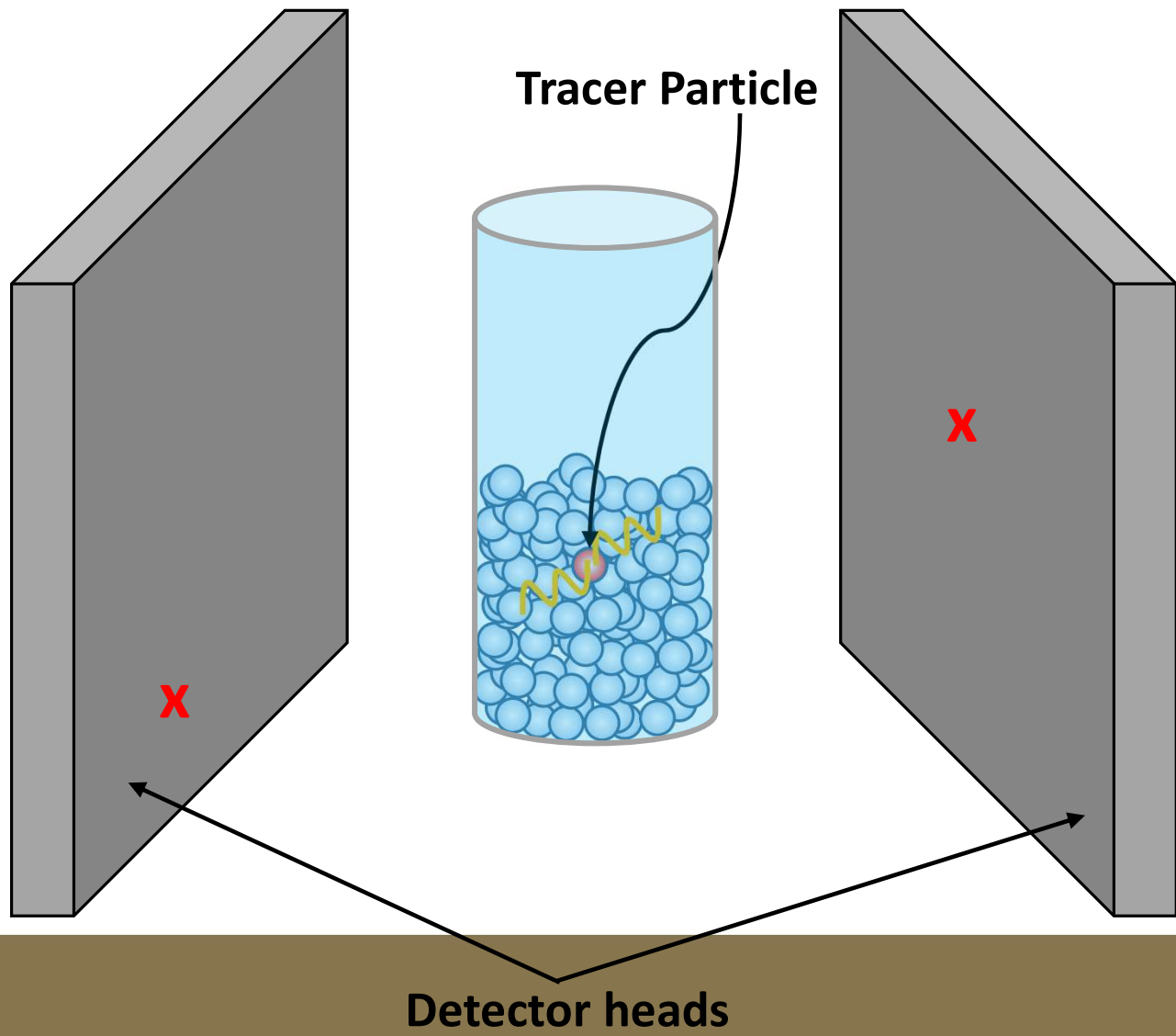
Uses highly-energetic gamma rays, capable of penetrating opaque media, including aluminium and steel

In essence, PEPT allows us to **‘see inside’ opaque systems**.



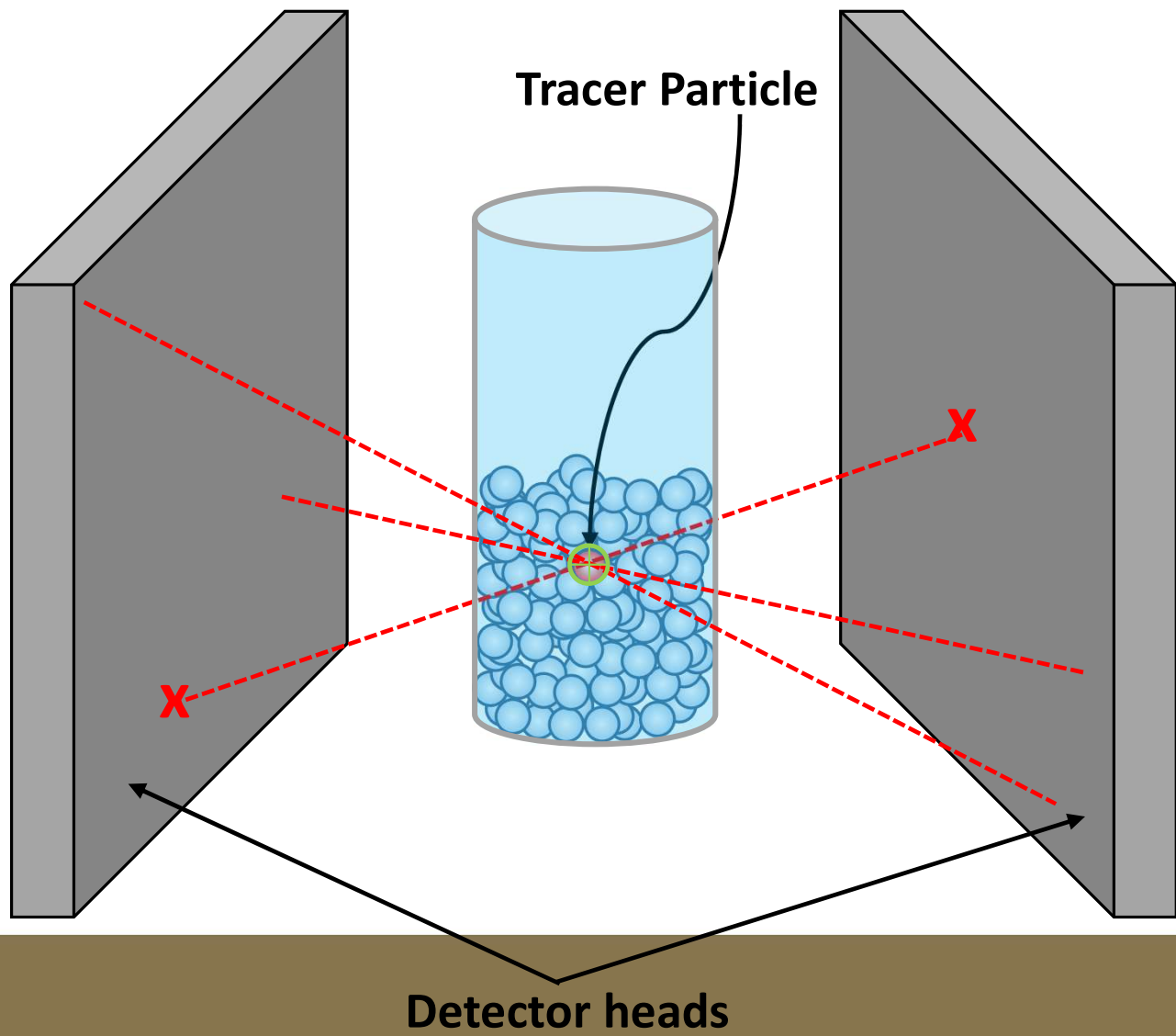


# How does it work?



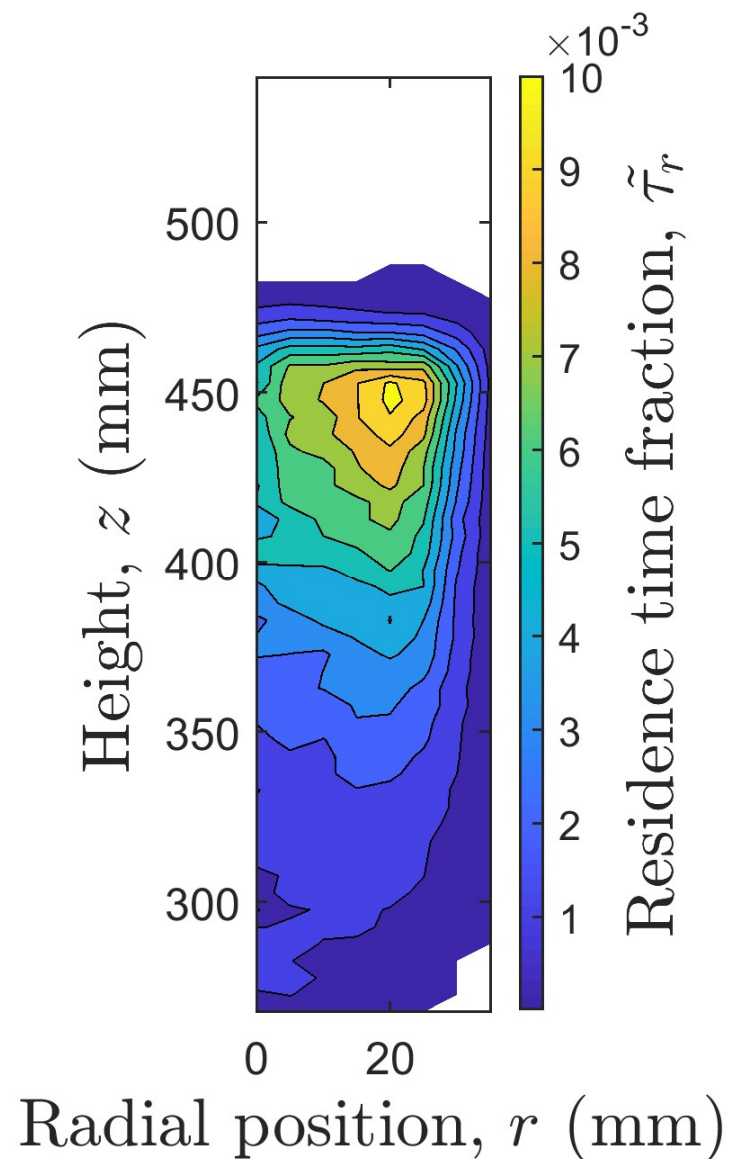
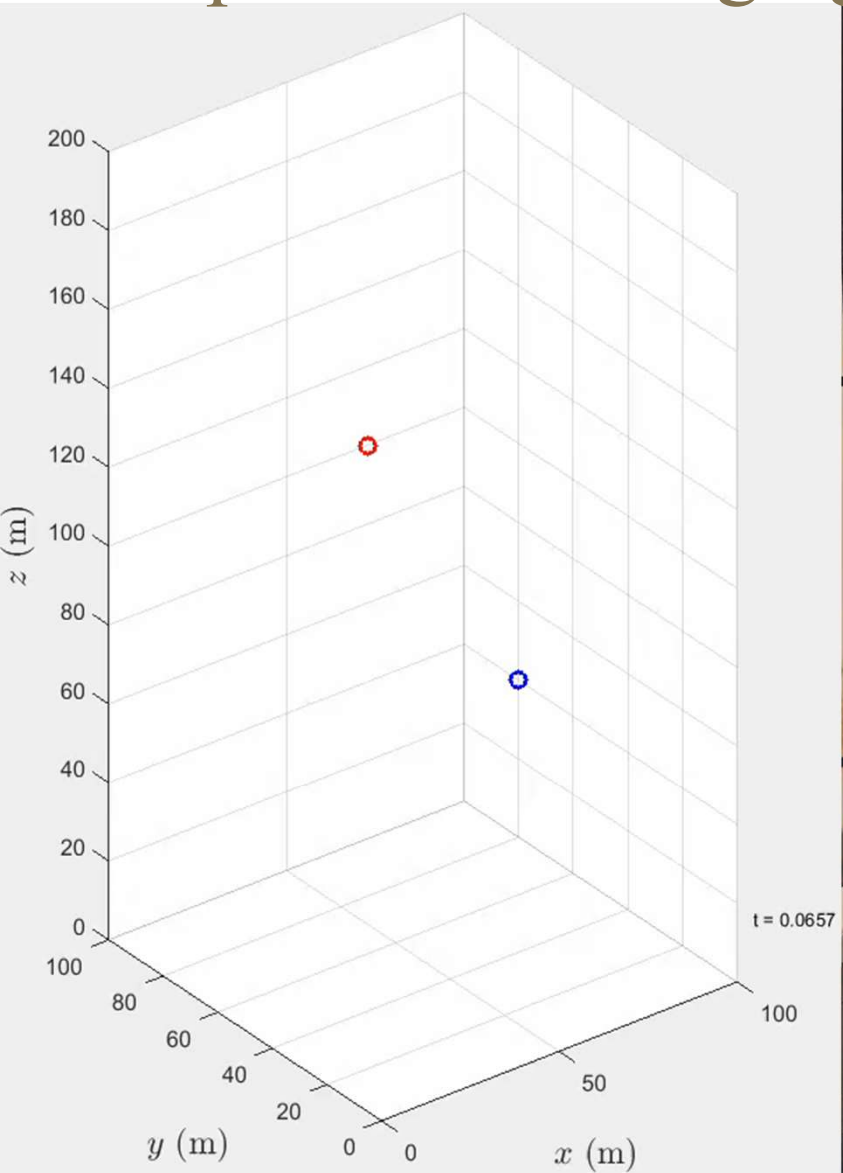
**Windows-Yule, C. R. K.,** Seville, J. P. K.,  
Ingram, A., & Parker, D. J. (2020).  
Positron Emission Particle Tracking of  
Granular Flows. *Annual Review of  
Chemical and Biomolecular  
Engineering*, 11.

# How does it work?

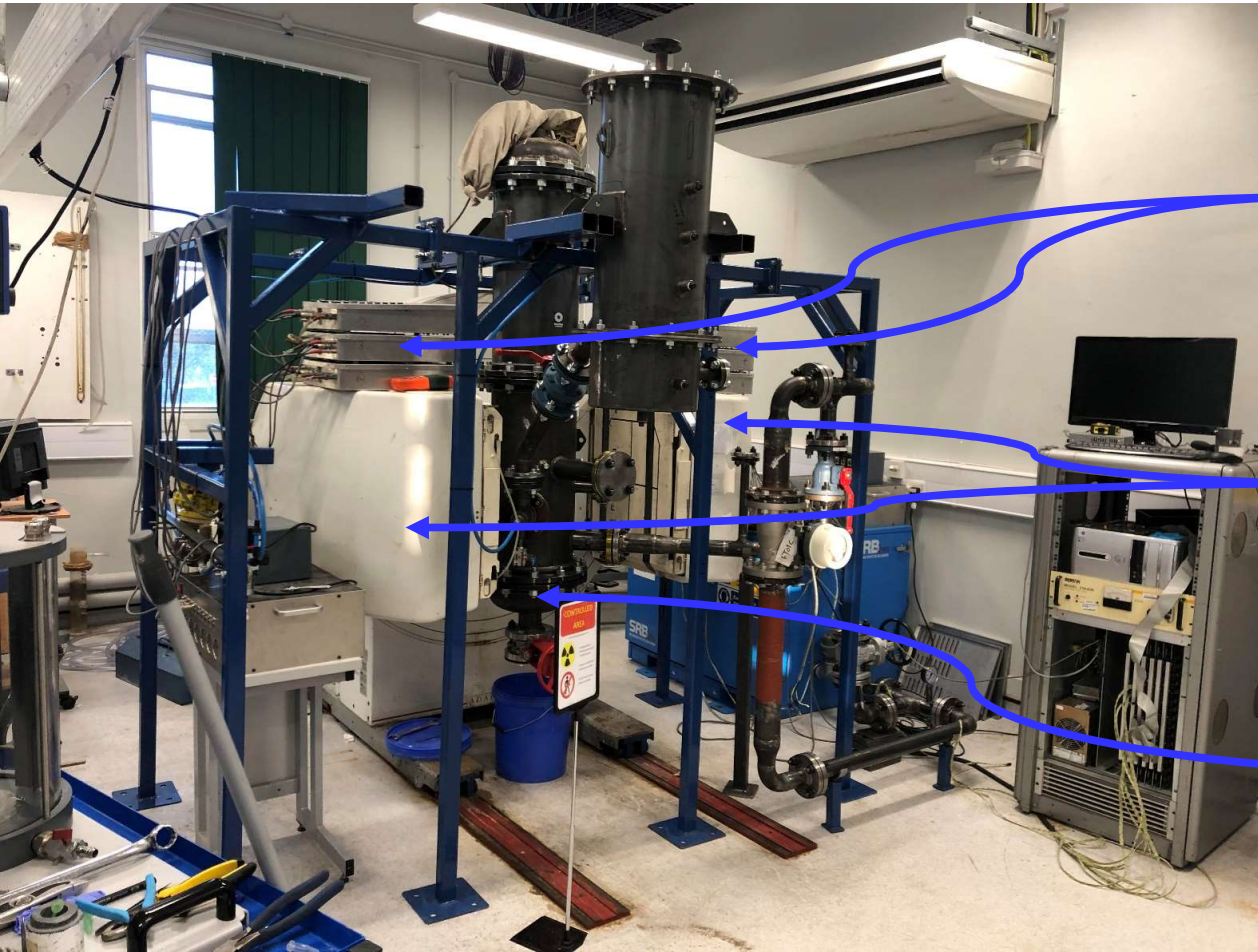


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Positron Emission Particle Tracking of  
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Chemical and Biomolecular  
Engineering*, 11.

# Example: PEPT imaging of a fluidised bed



# Example: PEPT imaging of a **serious** fluidised bed



Modular cameras provide additional, flexible imaging area

Main ADAC camera heads

Large, opaque vessel  
( $D = 300\text{mm}$ ,  $H > 1\text{m}$ )  
Solid steel walls



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# Example: PEPT imaging of a real, industrial fluidised bed



Featured in *Ingenia* magazine:



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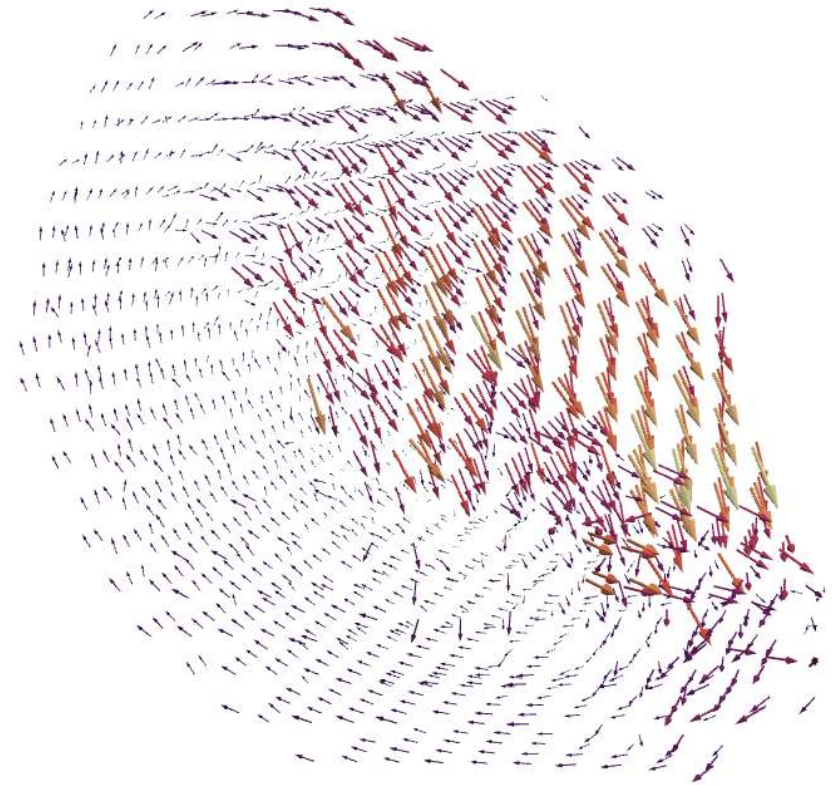
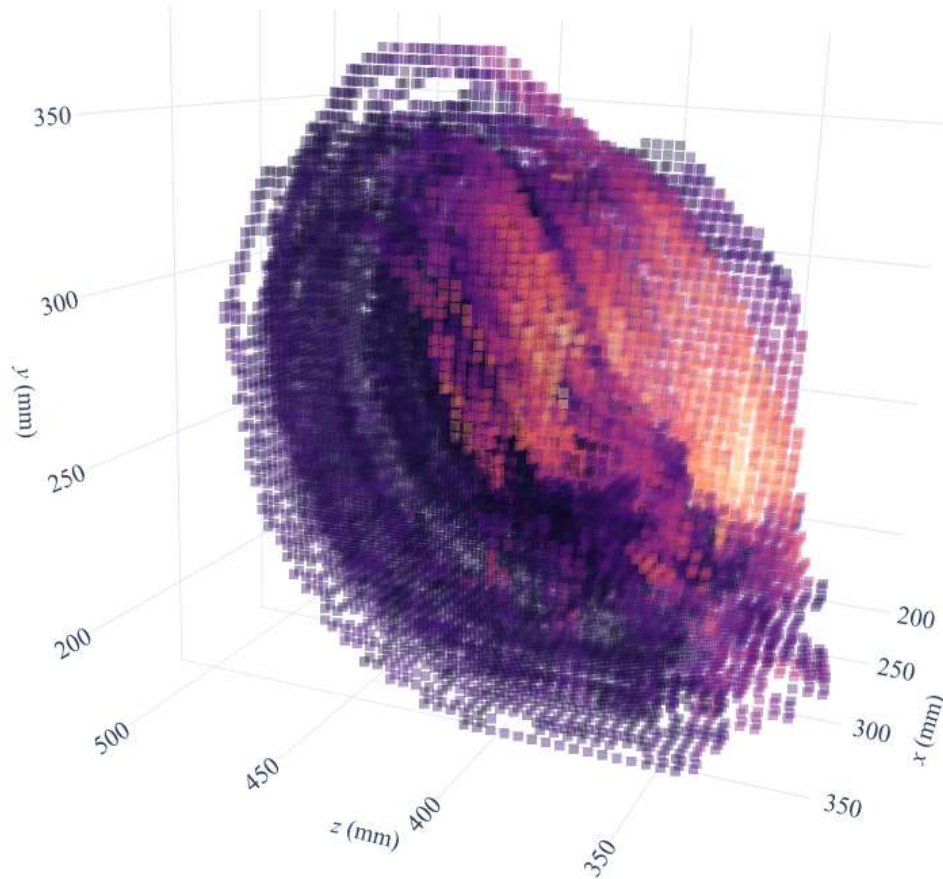
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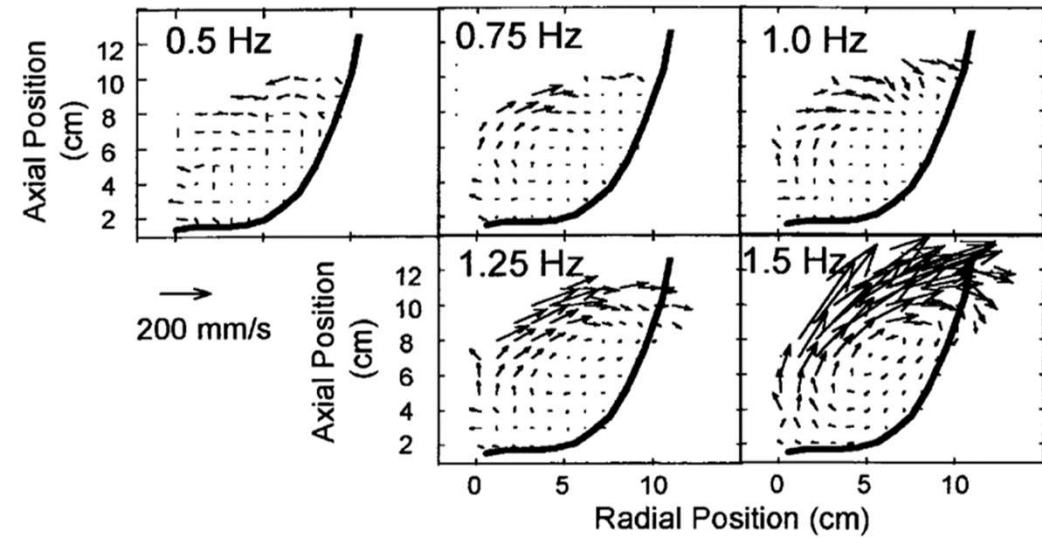
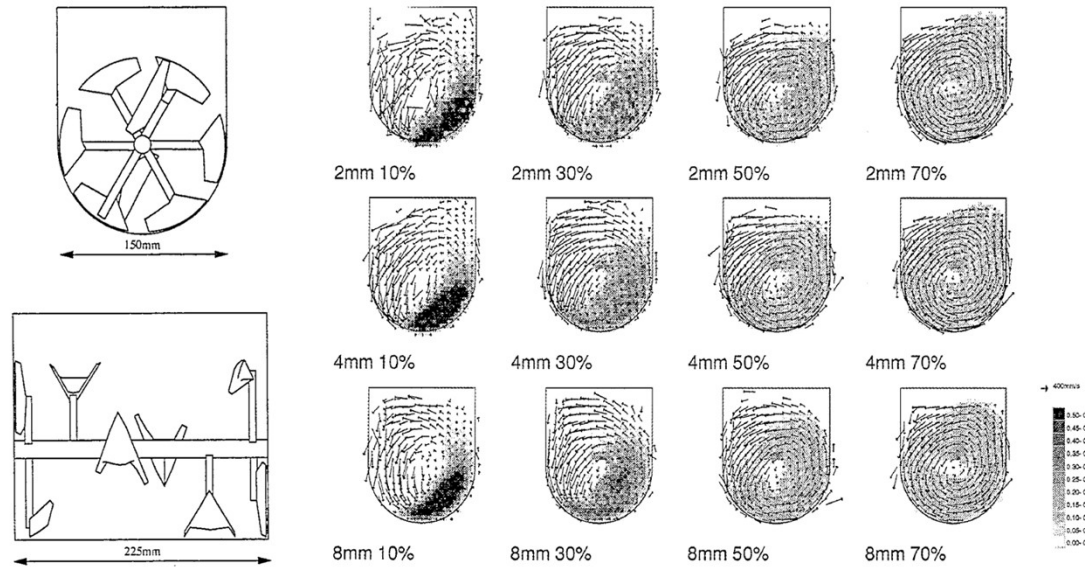


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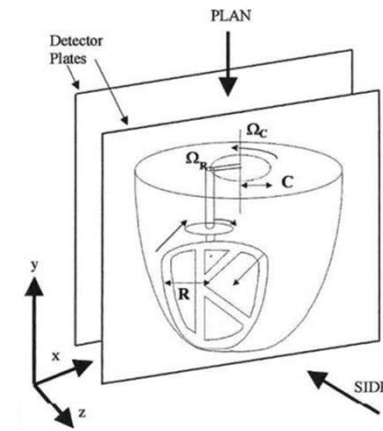
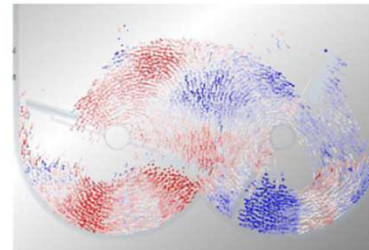
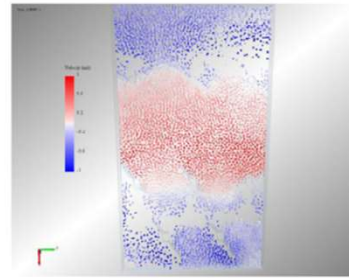
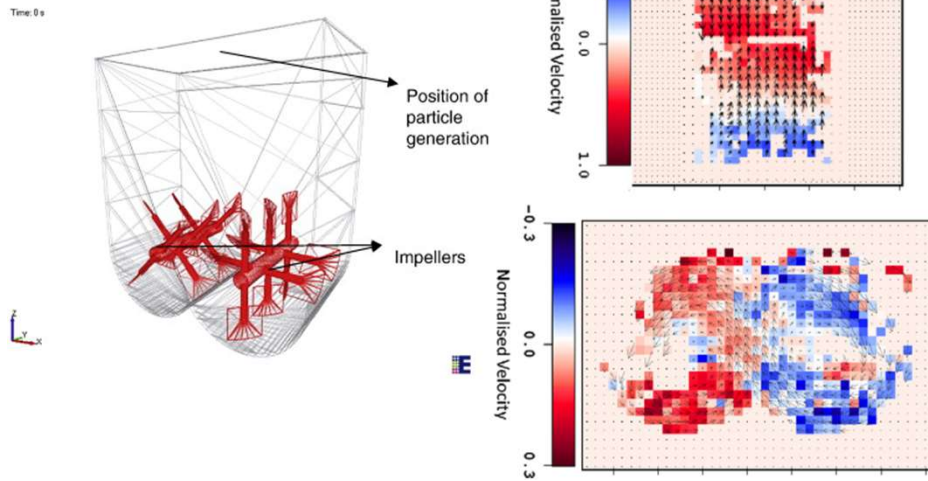
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# High-resolution, three-dimensional data

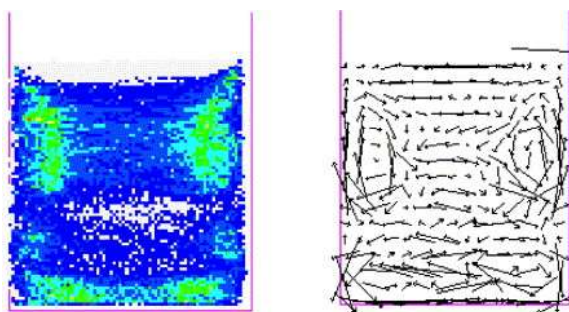
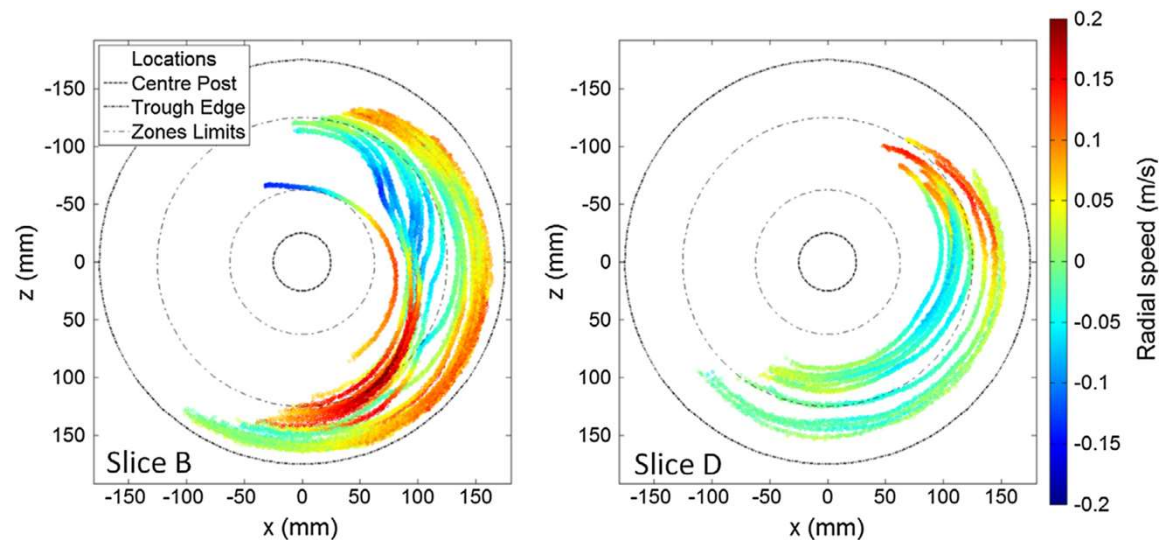
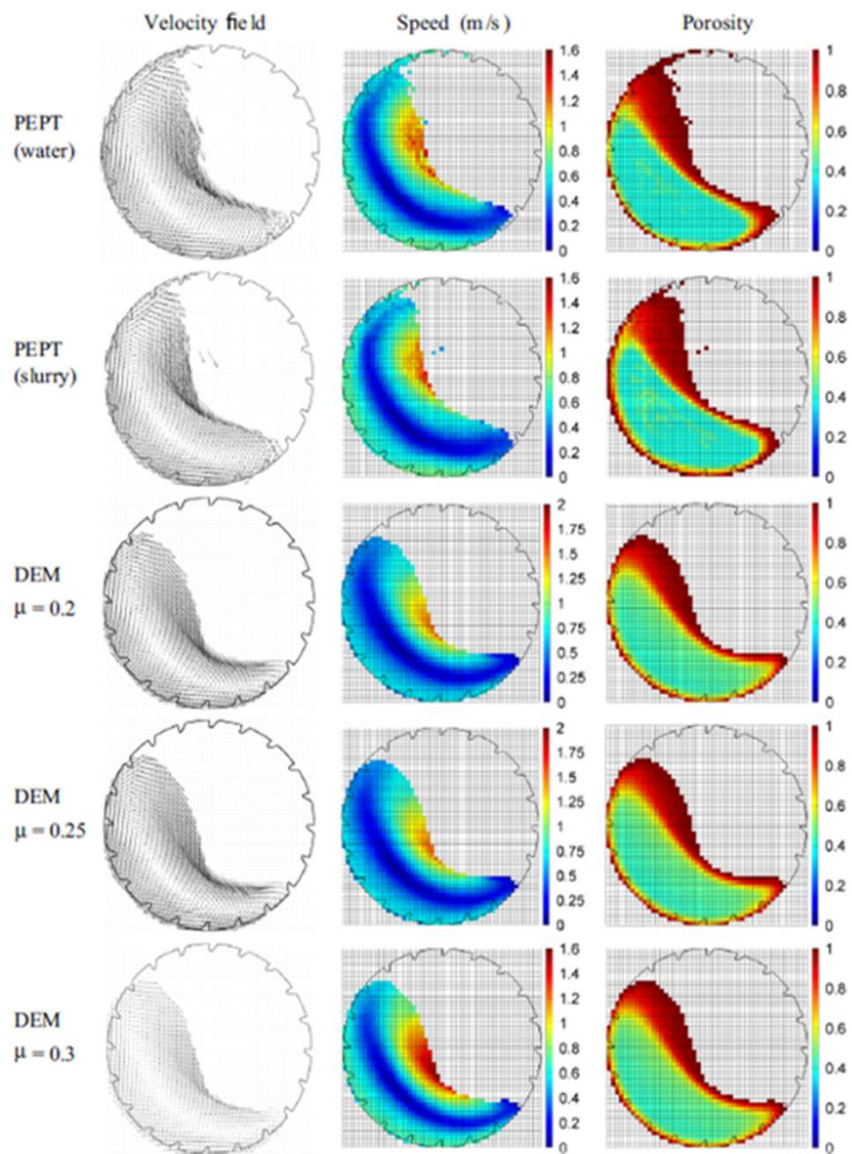




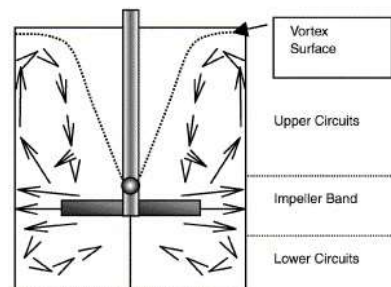
## Industrial Mixers



**Windows-Yule, C. R. K., Nicușan, A.L., Herald, M. T., Manger, S. & Parker, D.J., *PEPT, a Comprehensive Guide*, IoP Publishing, 2022**



Mills

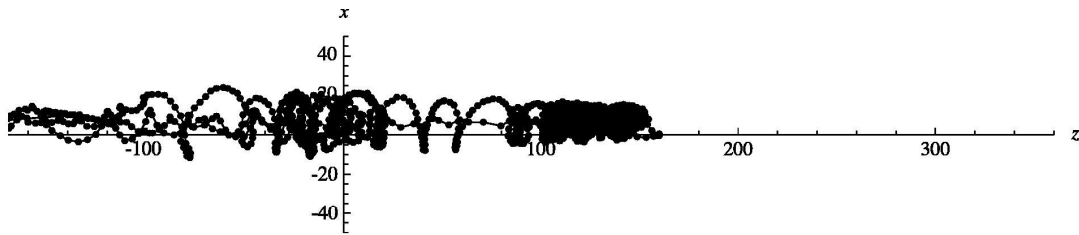
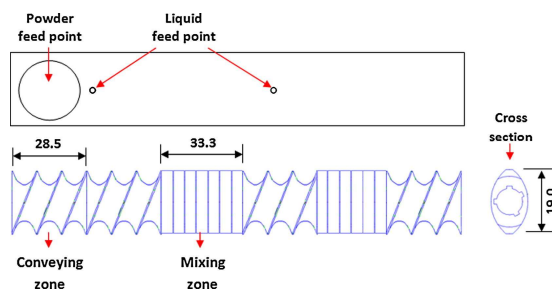
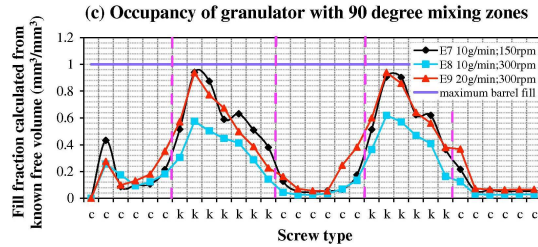
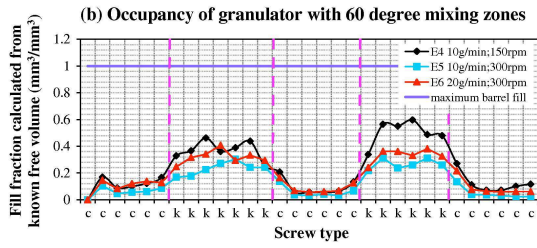
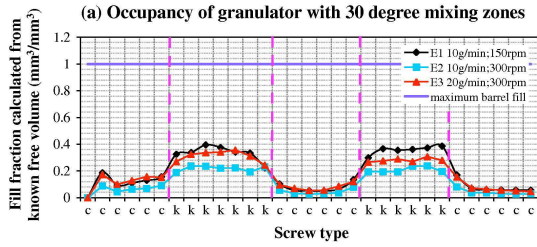


Spiral mineral concentrators

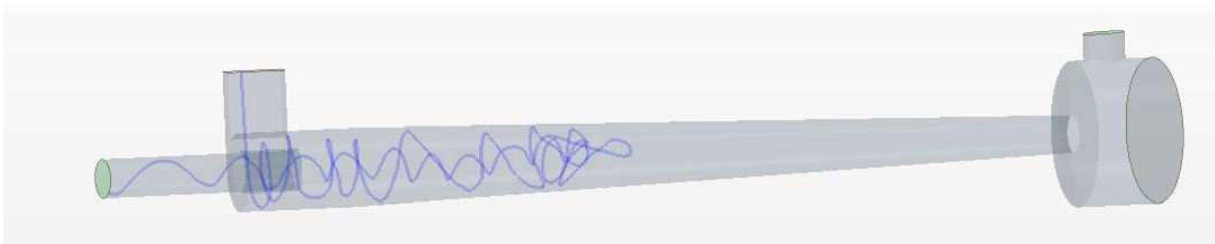
**Windows-Yule, C. R. K.,**  
 Nicuşan, A.L., Herald, M. T.,  
 Manger, S. & Parker, D.J., *PEPT,*  
*a Comprehensive Guide*, IoP  
 Publishing, 2022



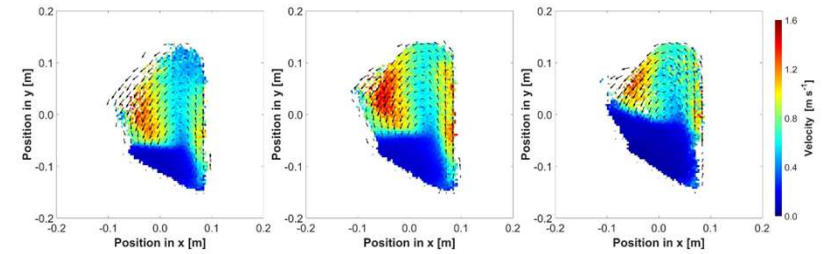
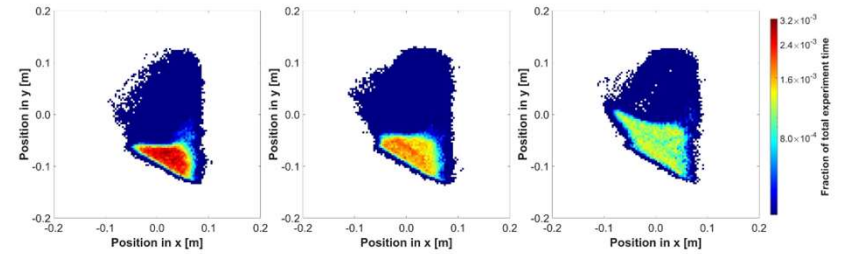
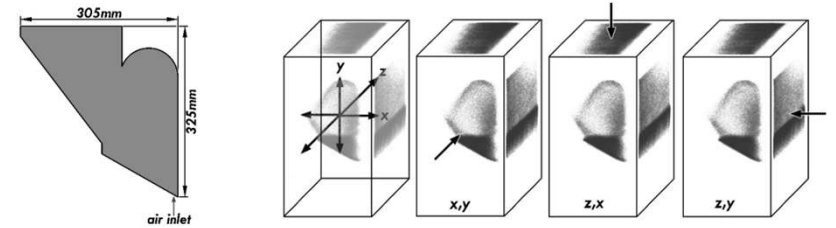
# Twin screw granulators



(a) Gas-/hydro-cyclones

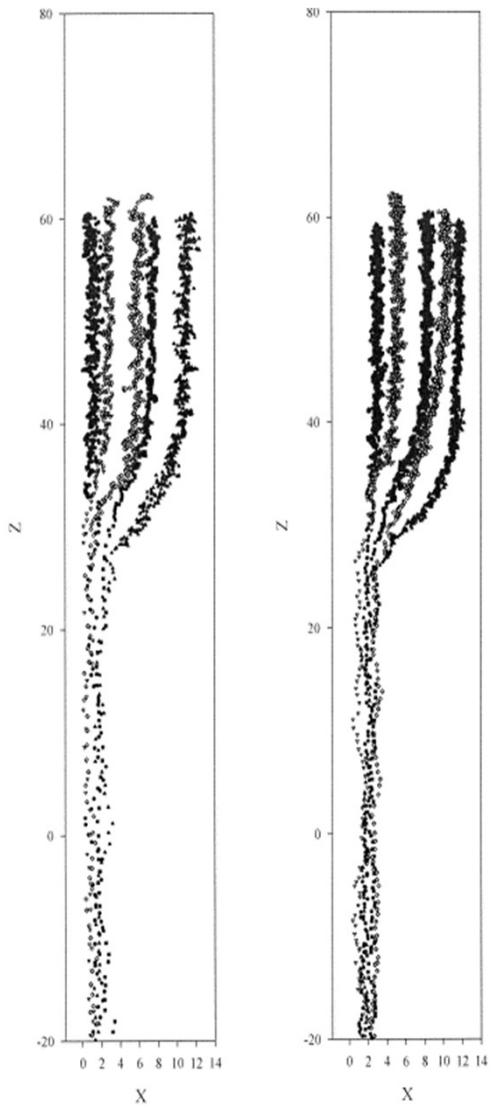


(b)

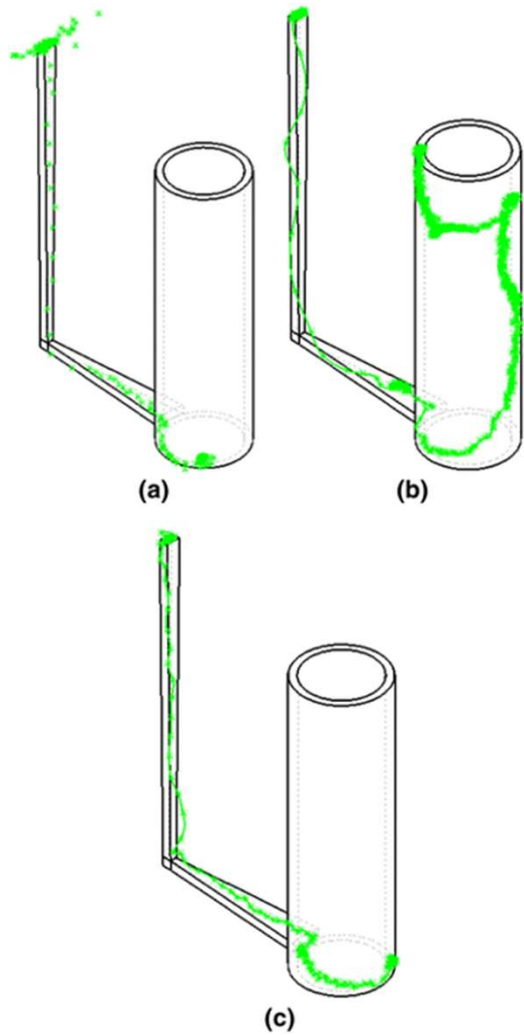


# Coffee roasters

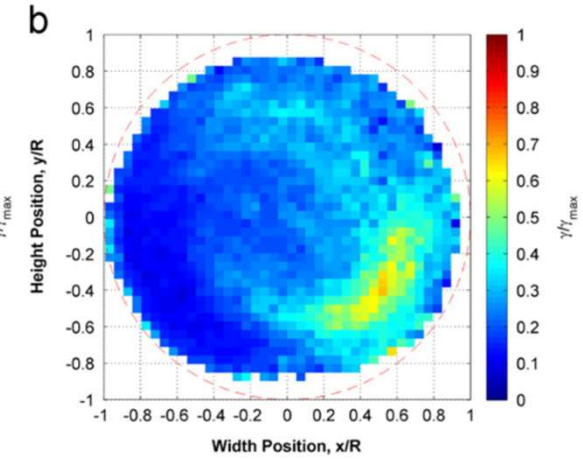
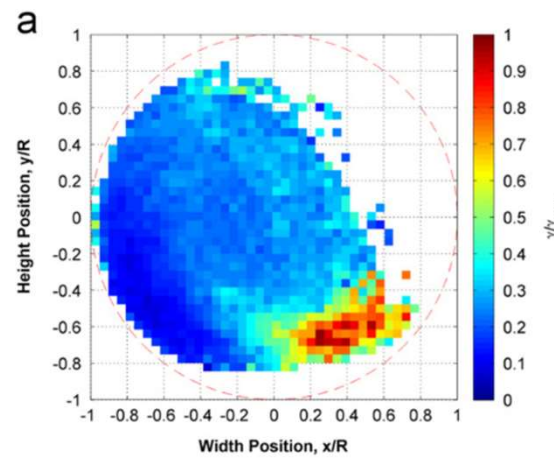
Windows-Yule, C. R. K., Nicuşan, A.L., Herald, M. T., Manger, S. & Parker, D.J., *PEPT, a Comprehensive Guide*, IoP Publishing, 2022



Extruders

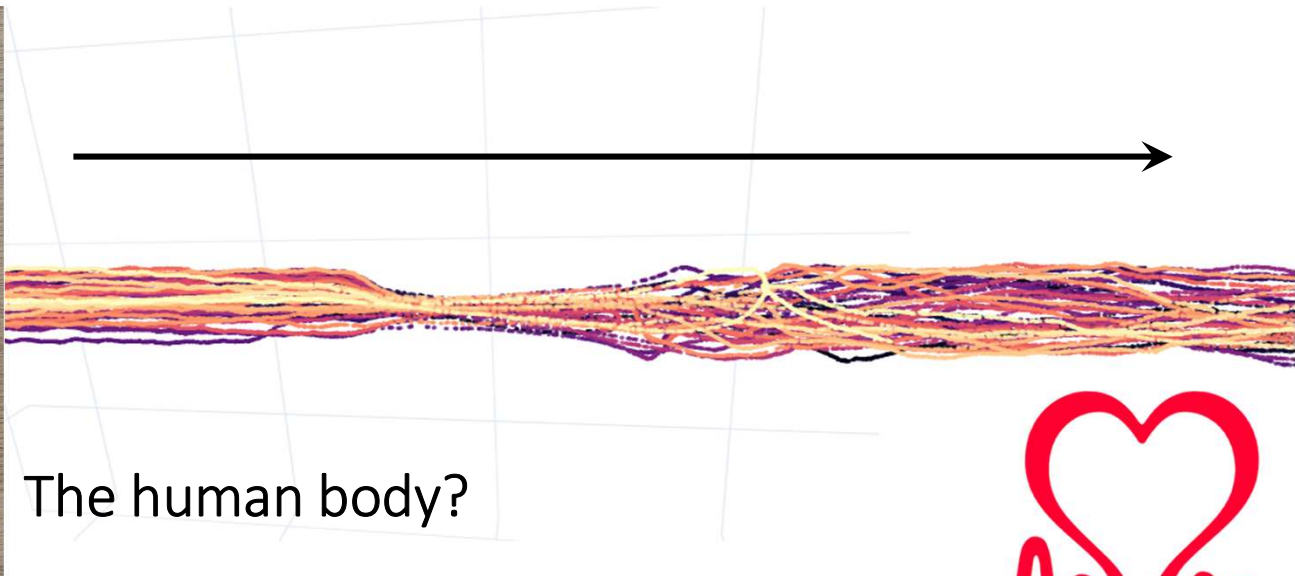


Metal casting

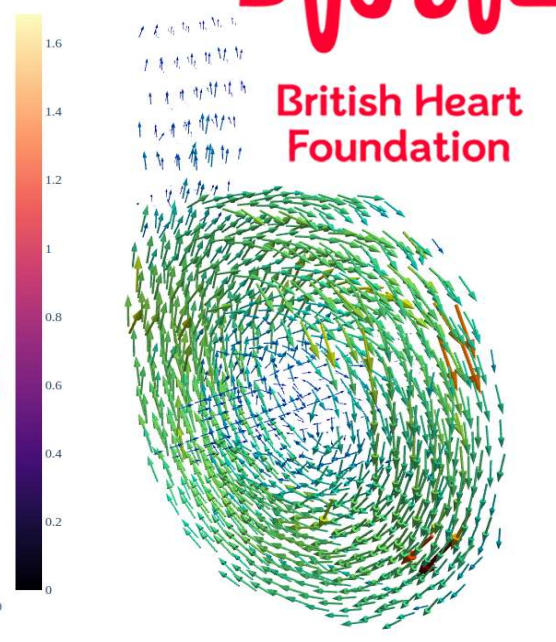
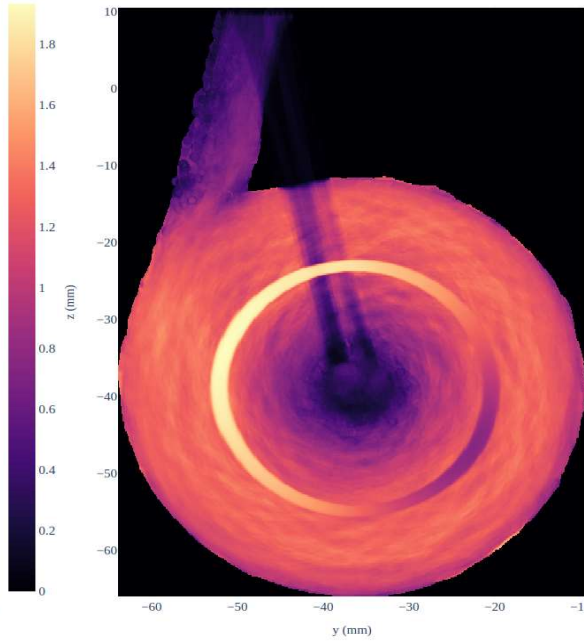
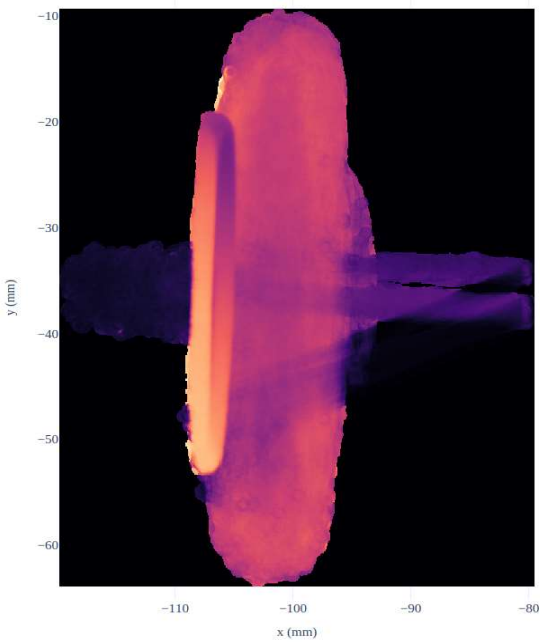
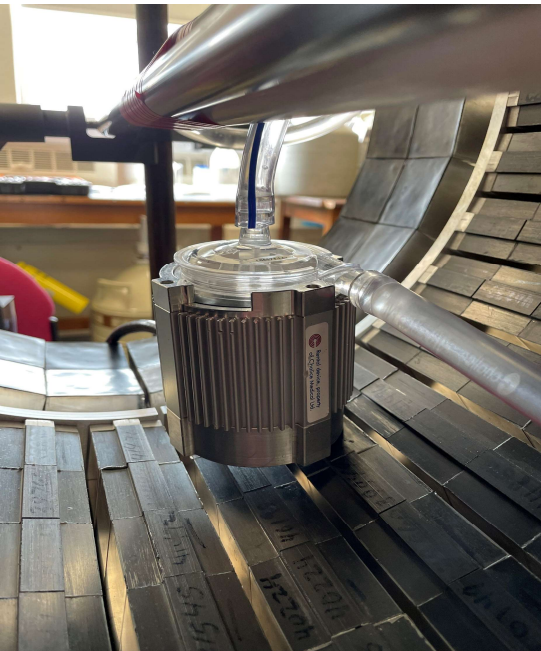


Domestic systems

Windows-Yule, C. R. K., Nicuşan, A.L., Herald, M. T., Manger, S. & Parker, D.J., *PEPT, a Comprehensive Guide*, IoP Publishing, 2022



The human body?



British Heart Foundation



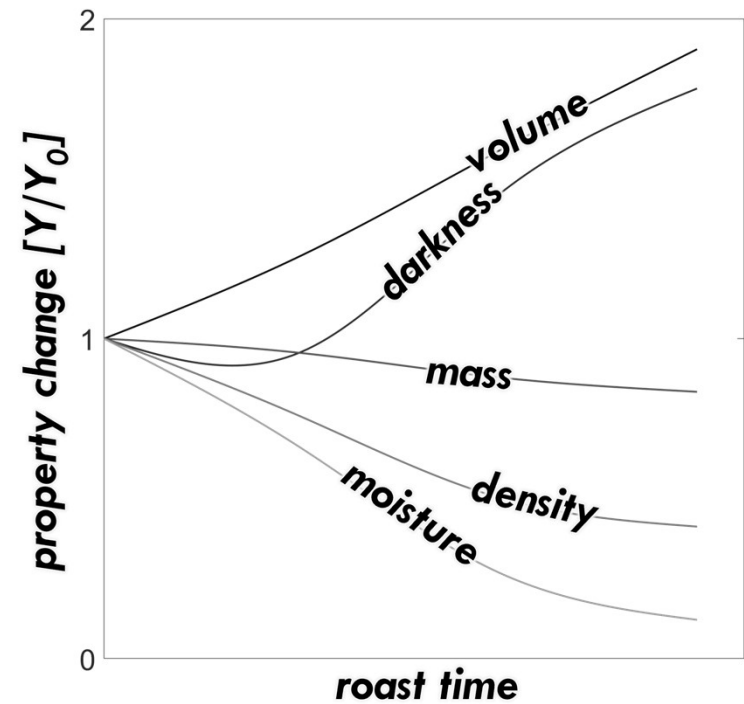
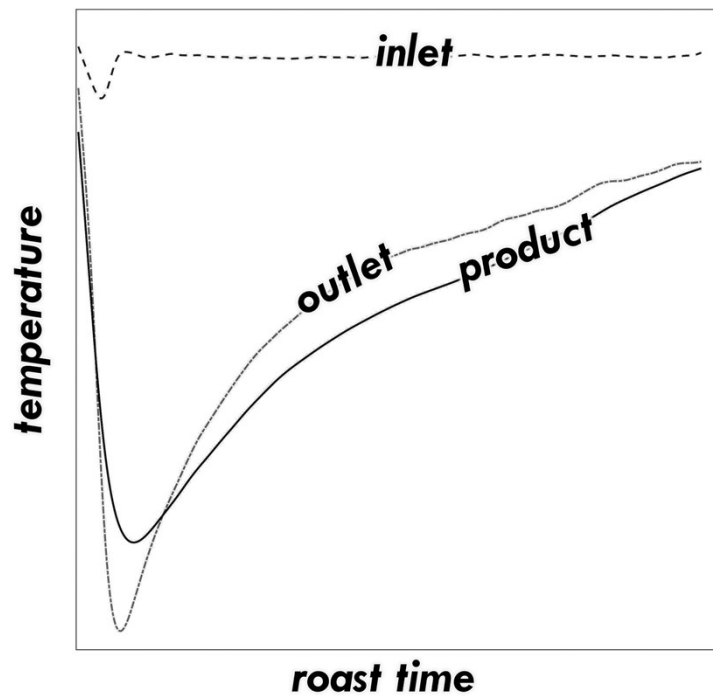
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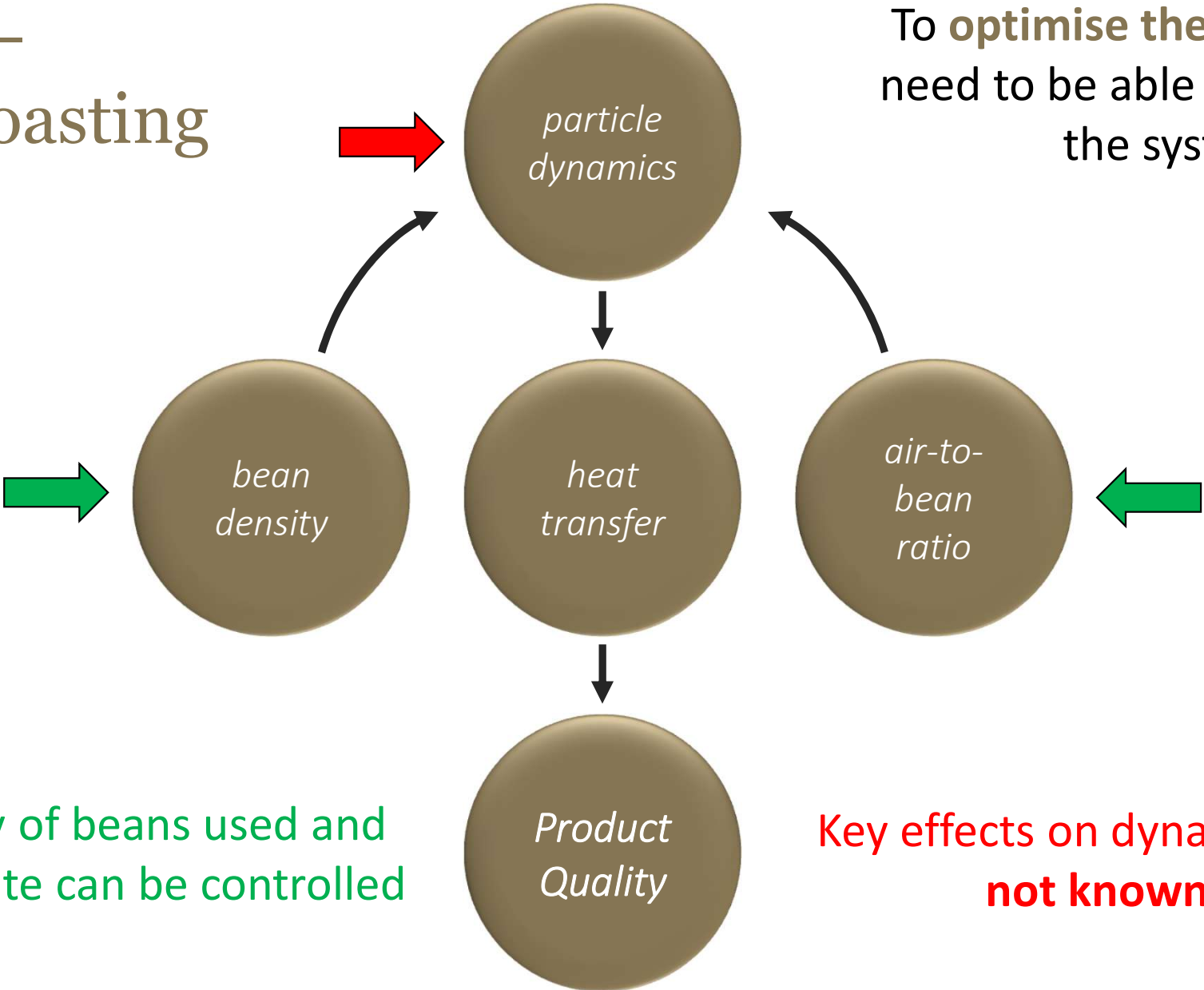
## II. Case Study: Spouted Bed Coffee Roaster (Jacobs Douwe Egberts)

# Context – Coffee Roasting

Transformations during roasting



# Context – Coffee Roasting



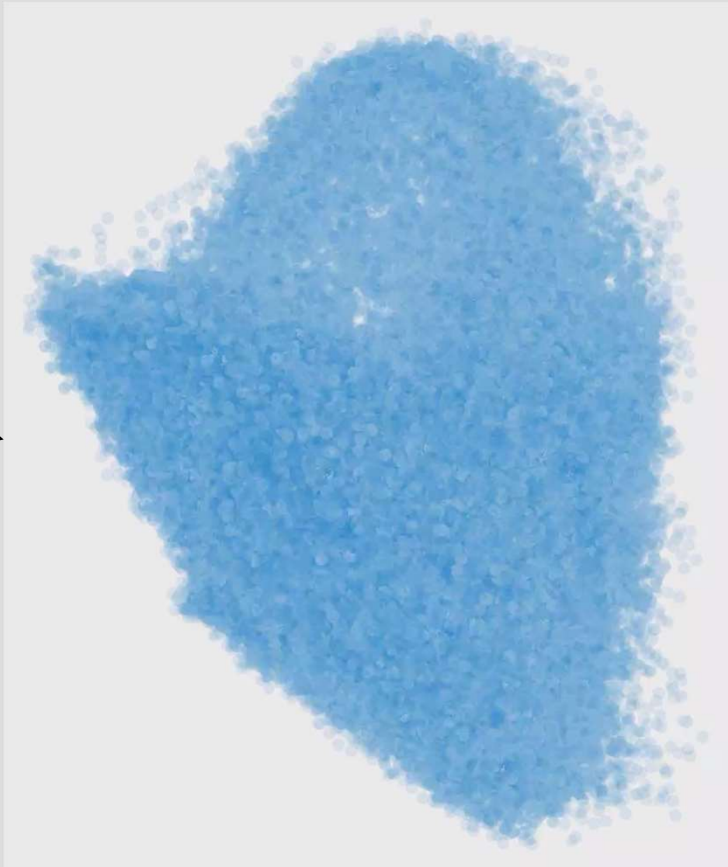
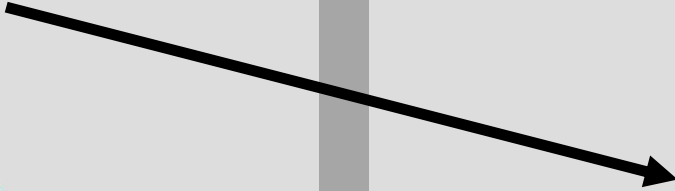
To **optimise the product** we need to be able to **see inside the system**

Quantity of beans used and airflow rate can be controlled

Key effects on dynamics are **not known**



# Experimental set-up





# Extracting Data from PEPT

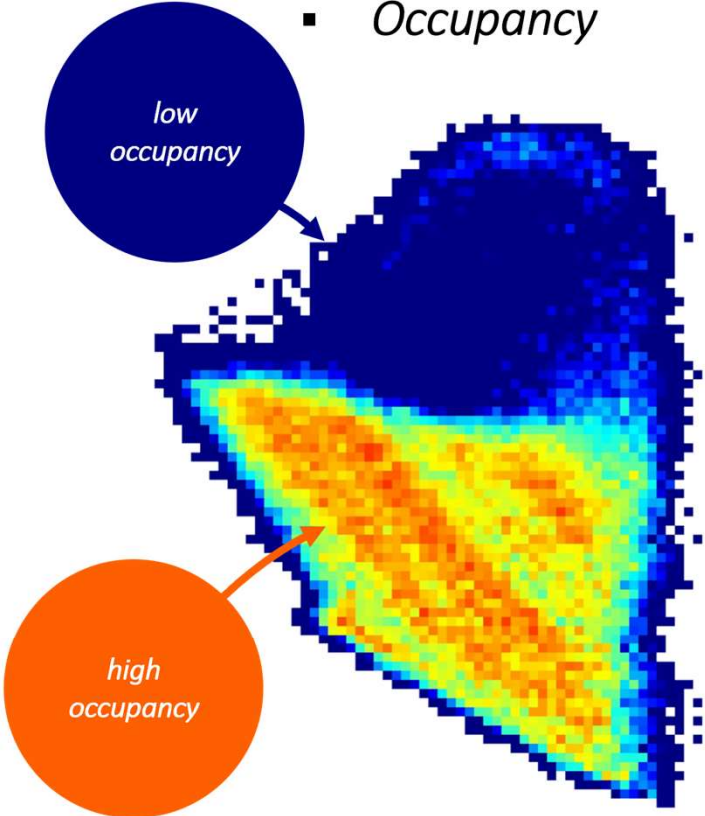
Al-Shemmeri, **Windows-Yule**, *et al.* (2021). Coffee bean particle motion in a spouted bed measured using Positron Emission Particle Tracking (PEPT). *Journal of Food Engineering*, 110709.

- *Trajectories*



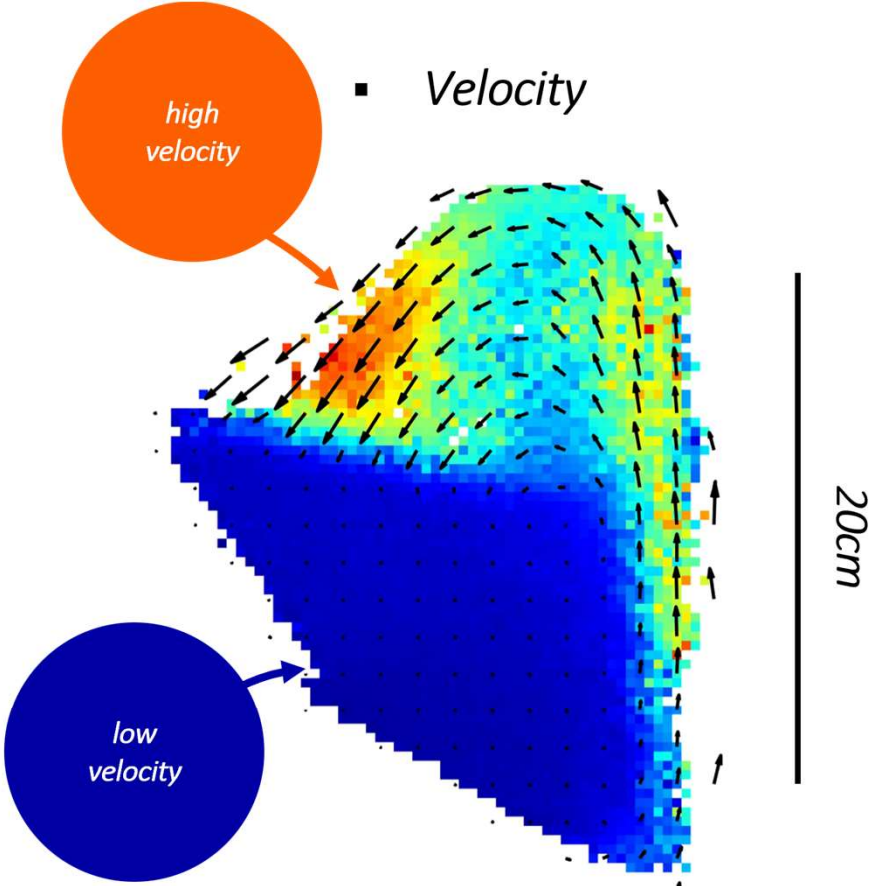
Trajectories identified via analysis of Cartesian co-ordinates (t,x,y,z)

- *Occupancy*



Fractional residence times determined using time-averaged Eulerian data

- *Velocity*

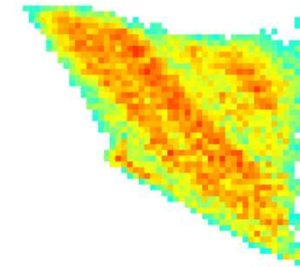
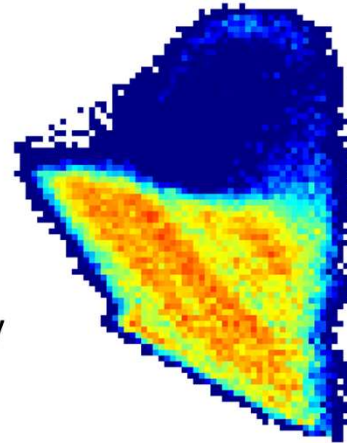


Velocity vectors determined using time-averaged Eulerian data

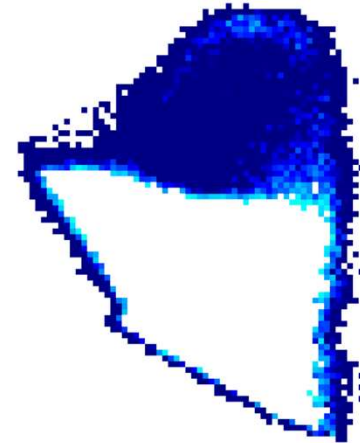
# Extracting Data from PEPT

Al-Shemmeri, **Windows-Yule**, *et al.* (2021). Coffee bean particle motion in a spouted bed measured using Positron Emission Particle Tracking (PEPT). *Journal of Food Engineering*, 110709.

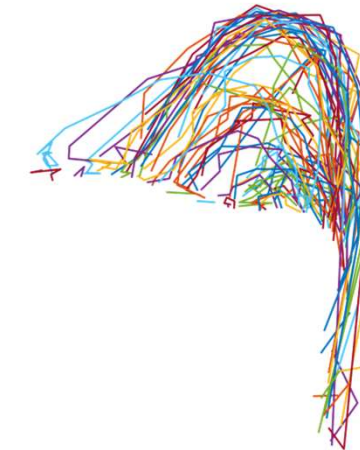
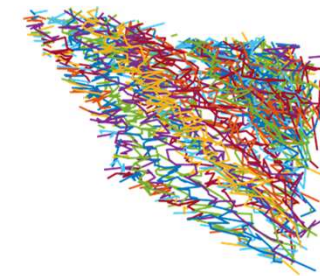
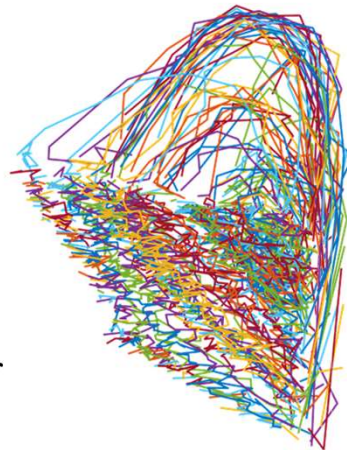
- Bean bed delineation
- Defined via Otsu method threshold
- Revealed Two distinct regions
  - (i) dense bean bed
    - Low velocity, high occupancy
    - Convective heat transfer limited
    - Lower temp. & heat transfer
  - (ii) dilute freeboard
    - high velocity, low occupancy
    - Convective heat transfer dominant
    - Higher temp. & heat transfer



(i) *dense bean bed*

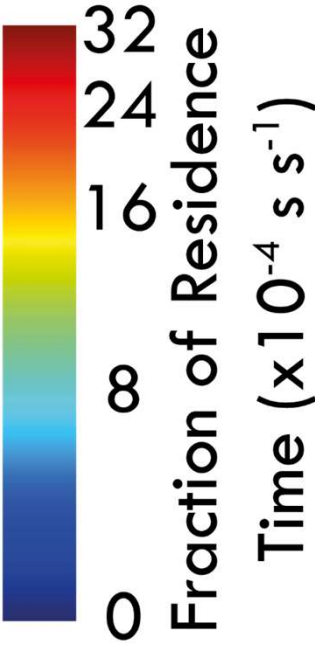
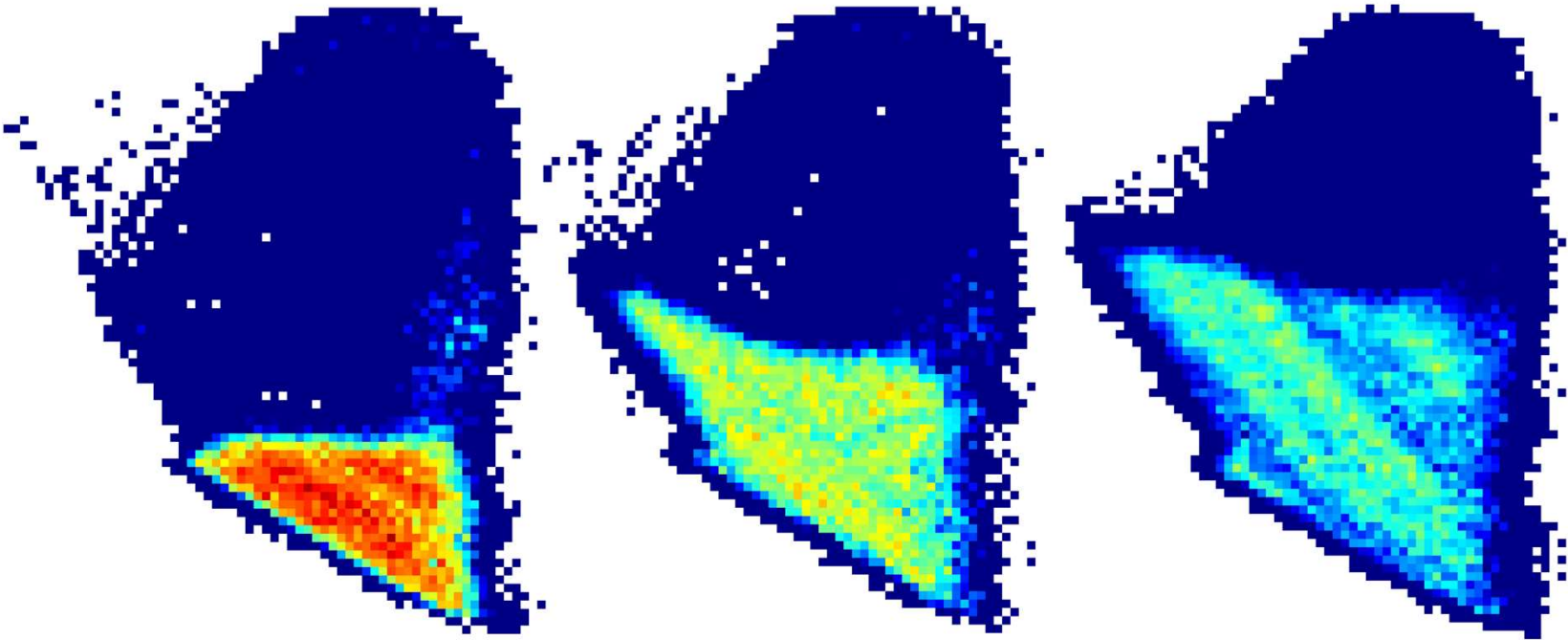


(i) *dilute freeboard*



# Results – Effect of batch size

Increasing batch size

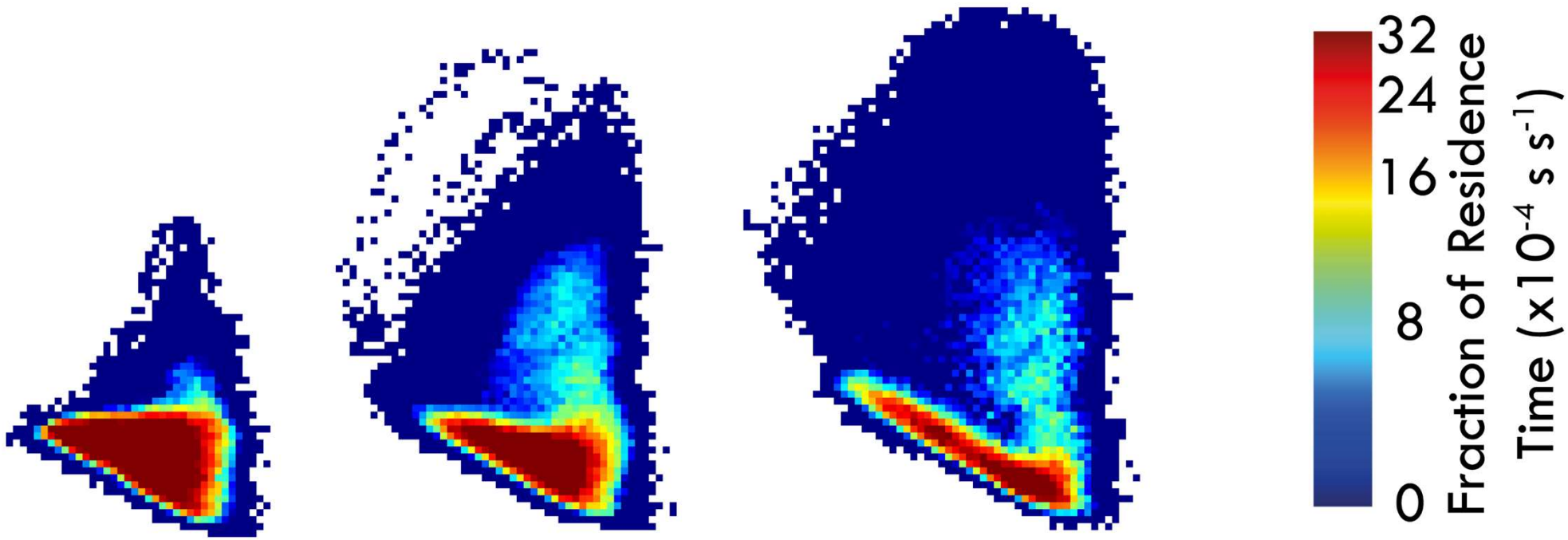


*roasted coffee with an air velocity of  $7.2 \text{ m s}^{-1}$*

Larger batch  $\rightarrow$  larger bed  $\rightarrow$  reduced heat transfer

# Results – Effect of batch size

Increasing air velocity



*200g batches of green coffee  
roasted coffee with an air velocity of 7.2 m s<sup>-1</sup>*

Higher air flow → smaller bed → improved heat transfer



# The Commercial Dilemma

## Larger batch size

- Increased throughput
- Decreased heat transfer

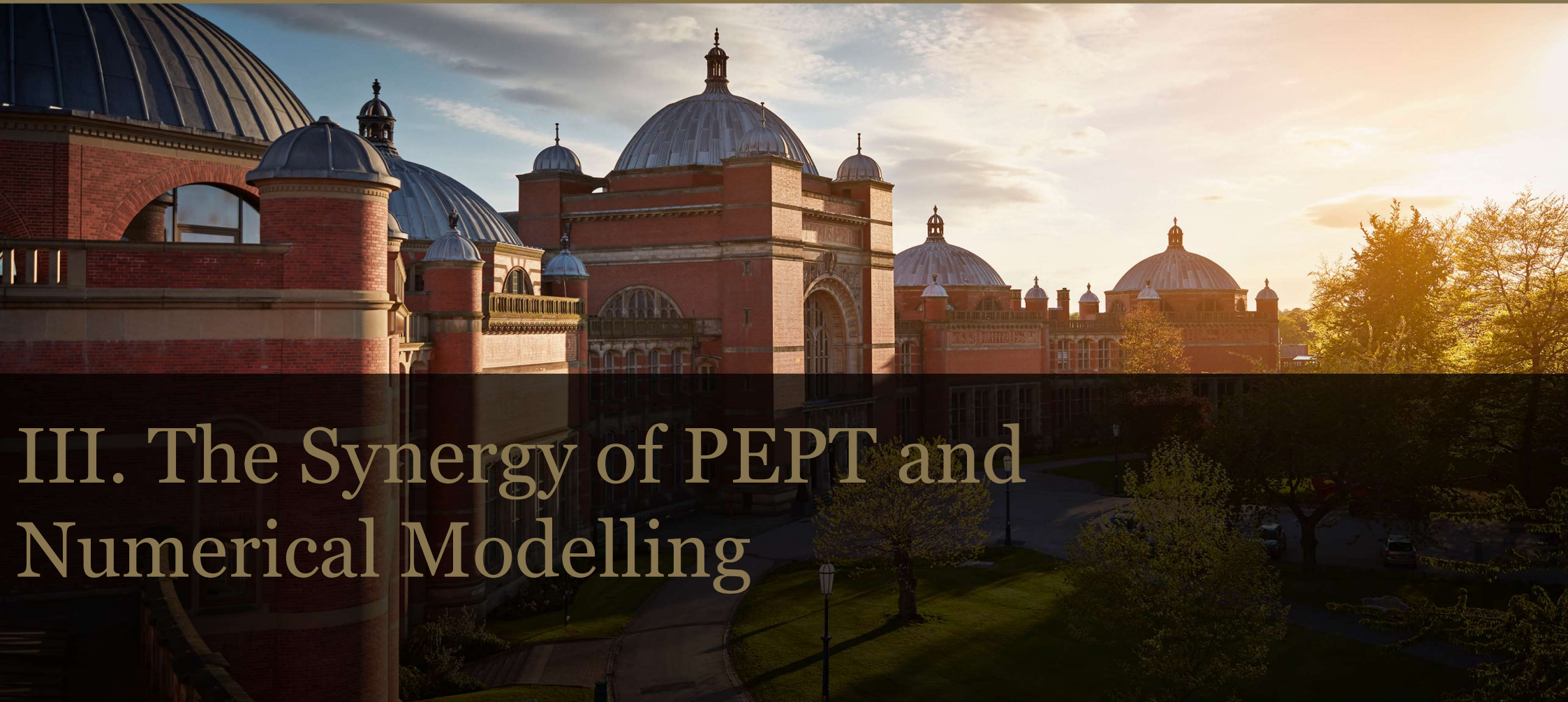
## Increased air flow

- Increased heat transfer
- Increased energy requirements

A complex optimization problem!



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# III. The Synergy of PEPT and Numerical Modelling



## An Efficient Route to Optimisation

- Solving an optimisation problem requires a **detailed exploration** of the relevant parameter space
- (Lots of experiments!)
- Though powerful, PEPT facilities are rare, and thus oversubscribed – and the technique is **costly to run**



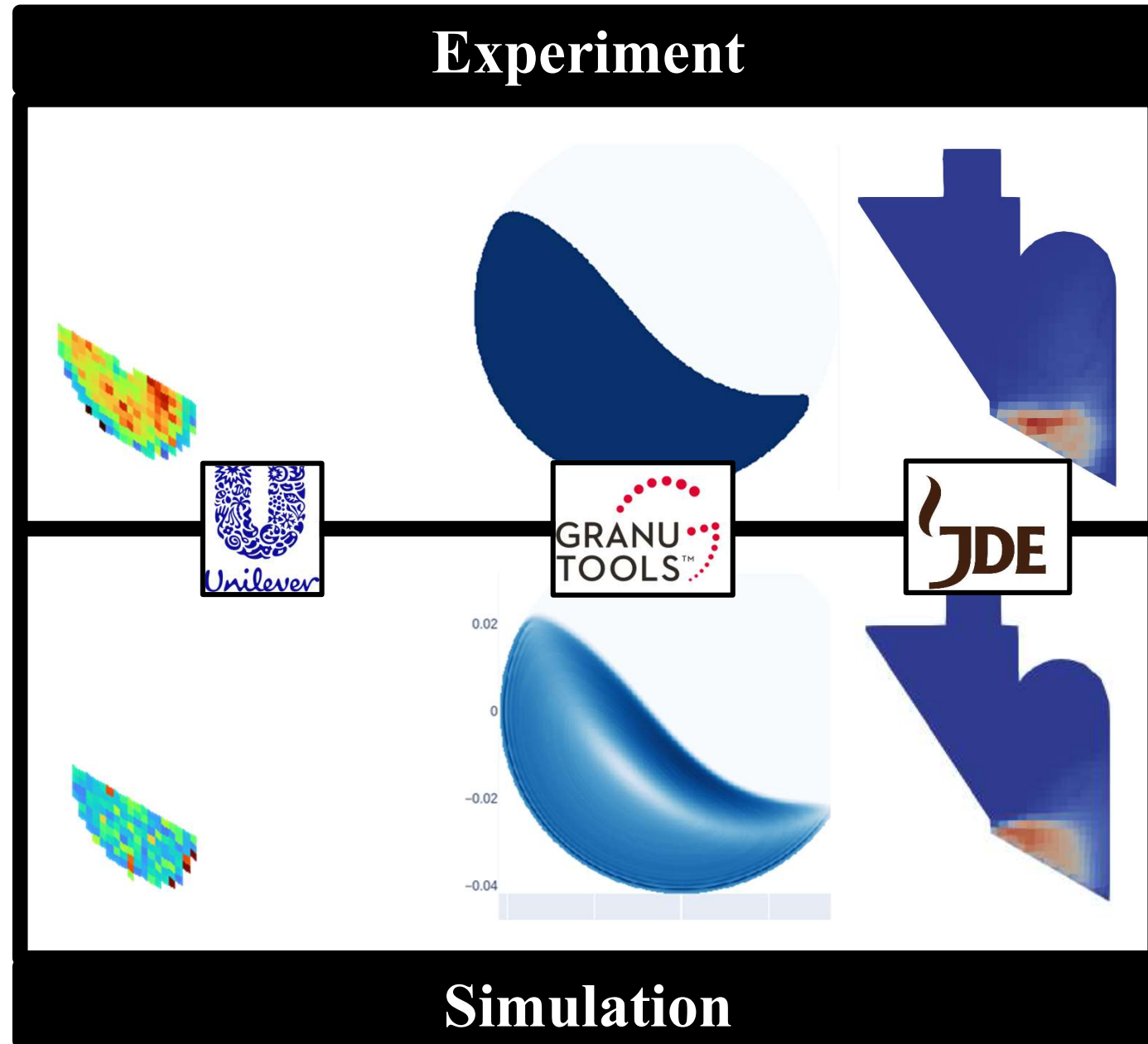
## An Efficient Route to Optimisation

- Numerical simulations – e.g. CFD-DEM – are **cheap to run** and thus allow the exploration of vast parameter space
- However, as established earlier, we **cannot trust them to be accurate**



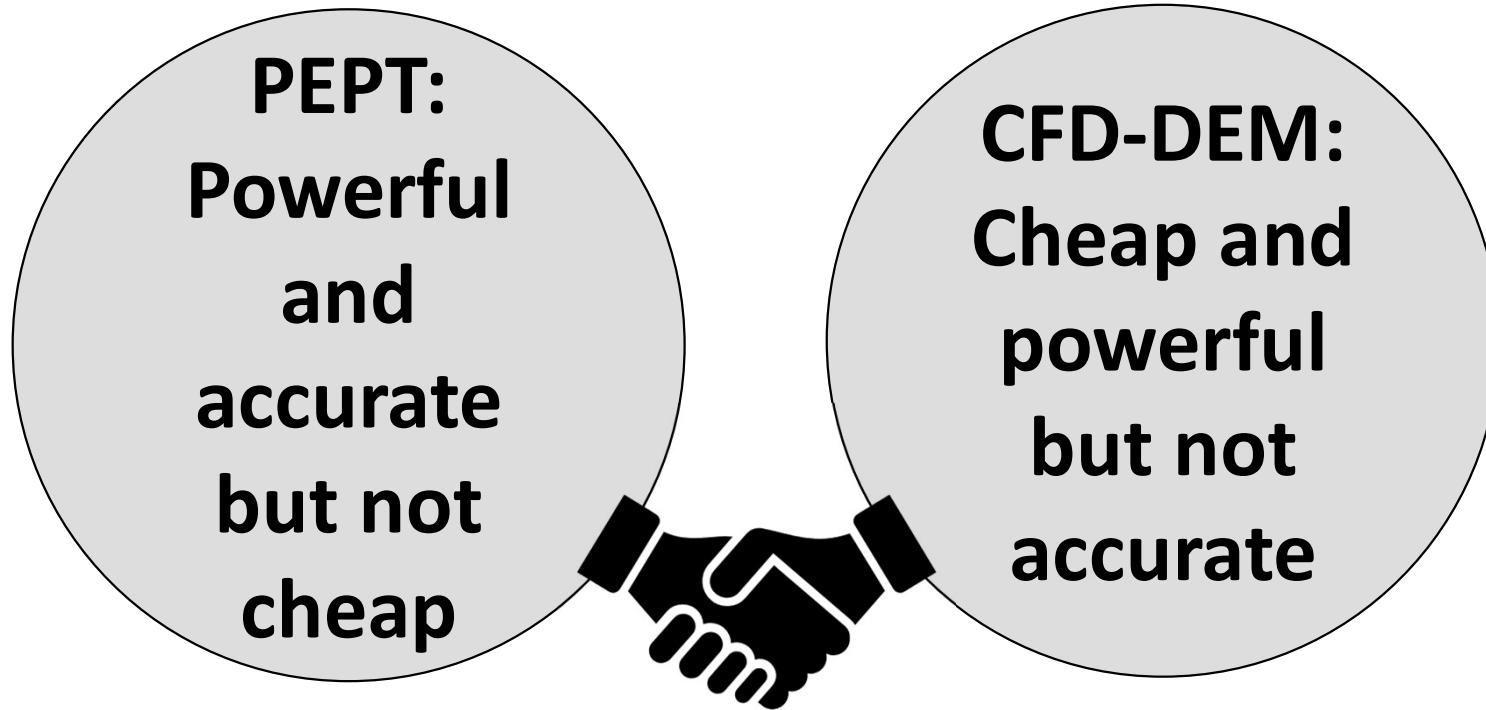
# PEPT as a Validation Tool

- PEPT can provide **identical outputs** to DEM, MP-PIC, CFD...
- → Facilitates **detailed, multi-point comparison** between experiment and simulation, considering **local variations** in key fields **at all points in space**
- → **Uniquely comprehensive validation**



**PEPT:  
Powerful  
and  
accurate  
but not  
cheap**

**CFD-DEM:  
Cheap and  
powerful  
but not  
accurate**

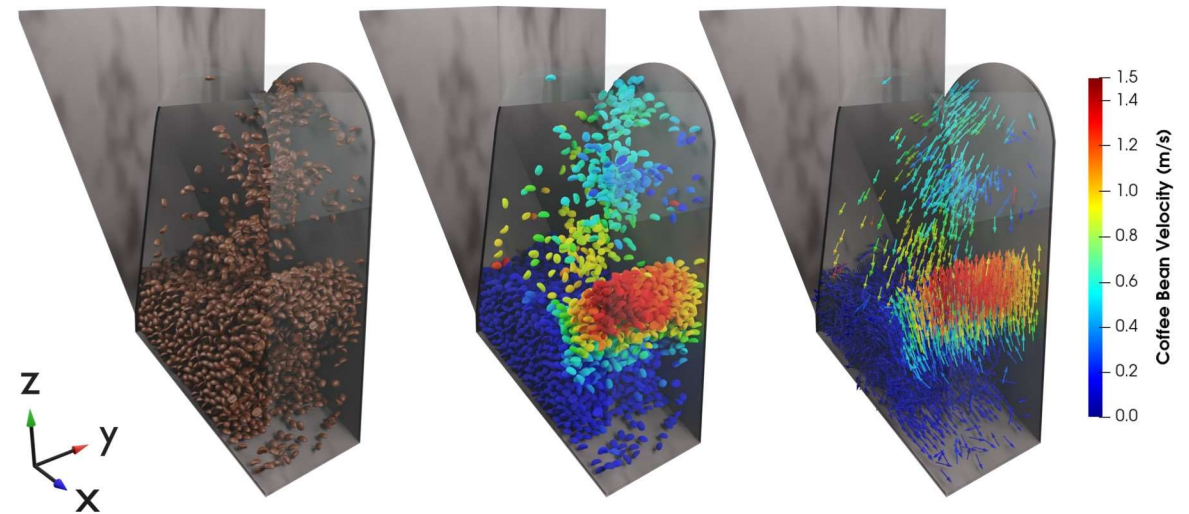


**PEPT + CFD-DEM: Powerful,  
accurate, and cost-effective**

# Back to our Case Study



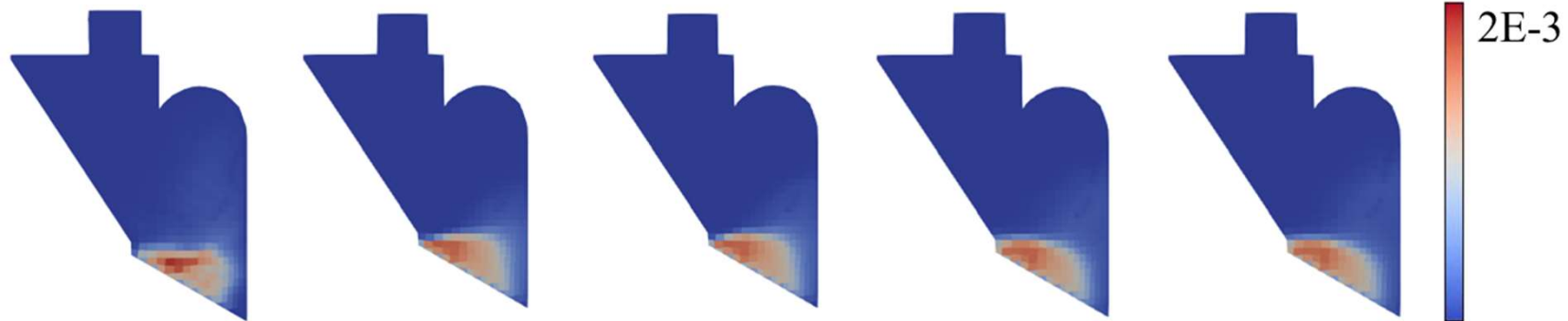
- CFD-DEM model of roaster
- 3D velocity & occupancy fields produced for both **PEPT** and **CFD-DEM**
- PEPT data and CFD-DEM data discretised on the same three-dimensional mesh



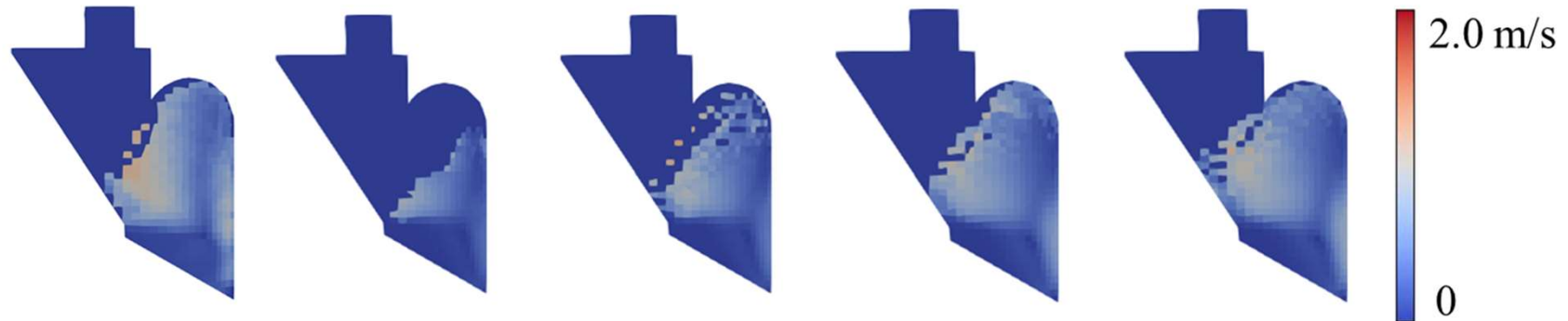
# Comparing PEPT & CFD-DEM

Che, **Windows-Yule**, et al. (2023). PEPT validated CFD-DEM model of aspherical particle motion in a spouted bed. *Chemical Engineering Journal*, 453, 139689.

Occupancy:



Solid velocity:



PEPT

CFD-DEM  
Model 1

CFD-DEM  
Model 2

CFD-DEM  
Model 3

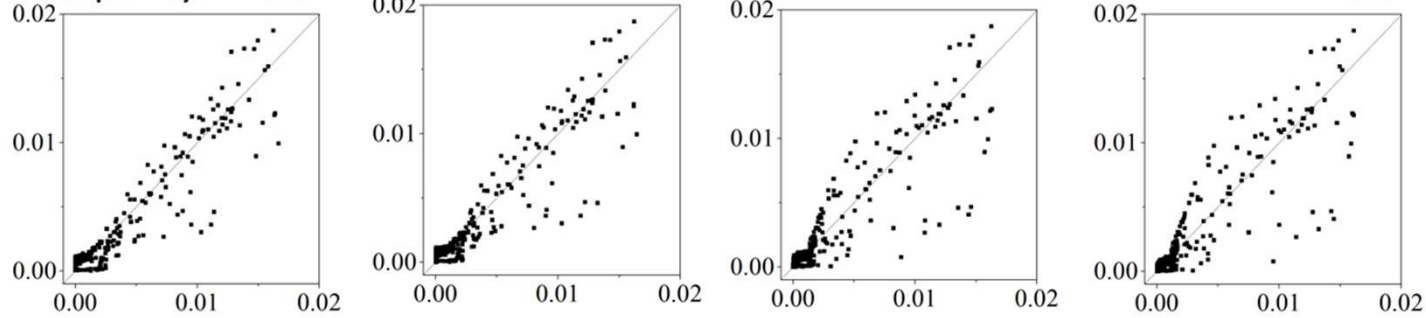
CFD-DEM  
Model 4

Cell-by-cell comparison of multiple three-dimensional fields → **detailed, highly-rigorous validation** of models used → aid model choice.

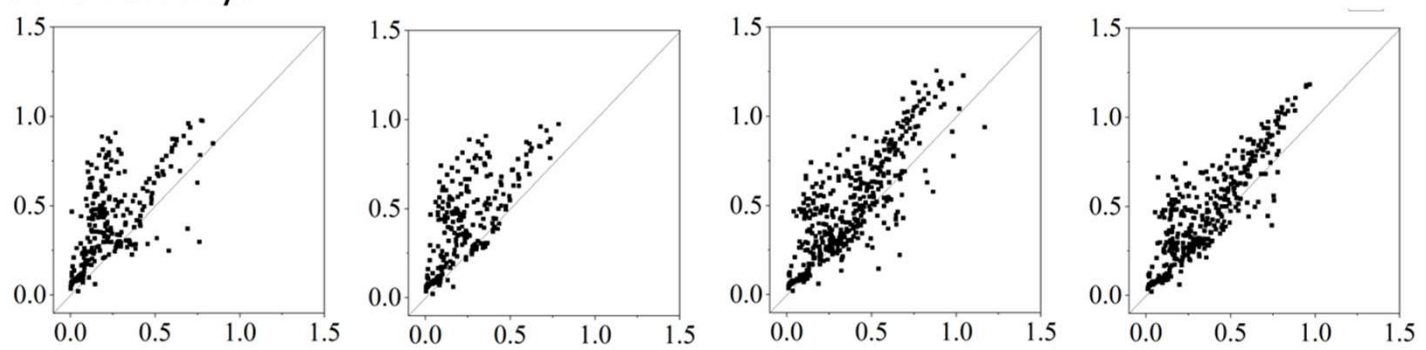
# Rigorous, Quantitative Validation

Che, **Windows-Yule**, et al. (2023). PEPT validated CFD-DEM model of aspherical particle motion in a spouted bed. *Chemical Engineering Journal*, 453, 139689.

Occupancy:



Solid velocity:



CFD-DEM  
Model 1

CFD-DEM  
Model 2

CFD-DEM  
Model 3

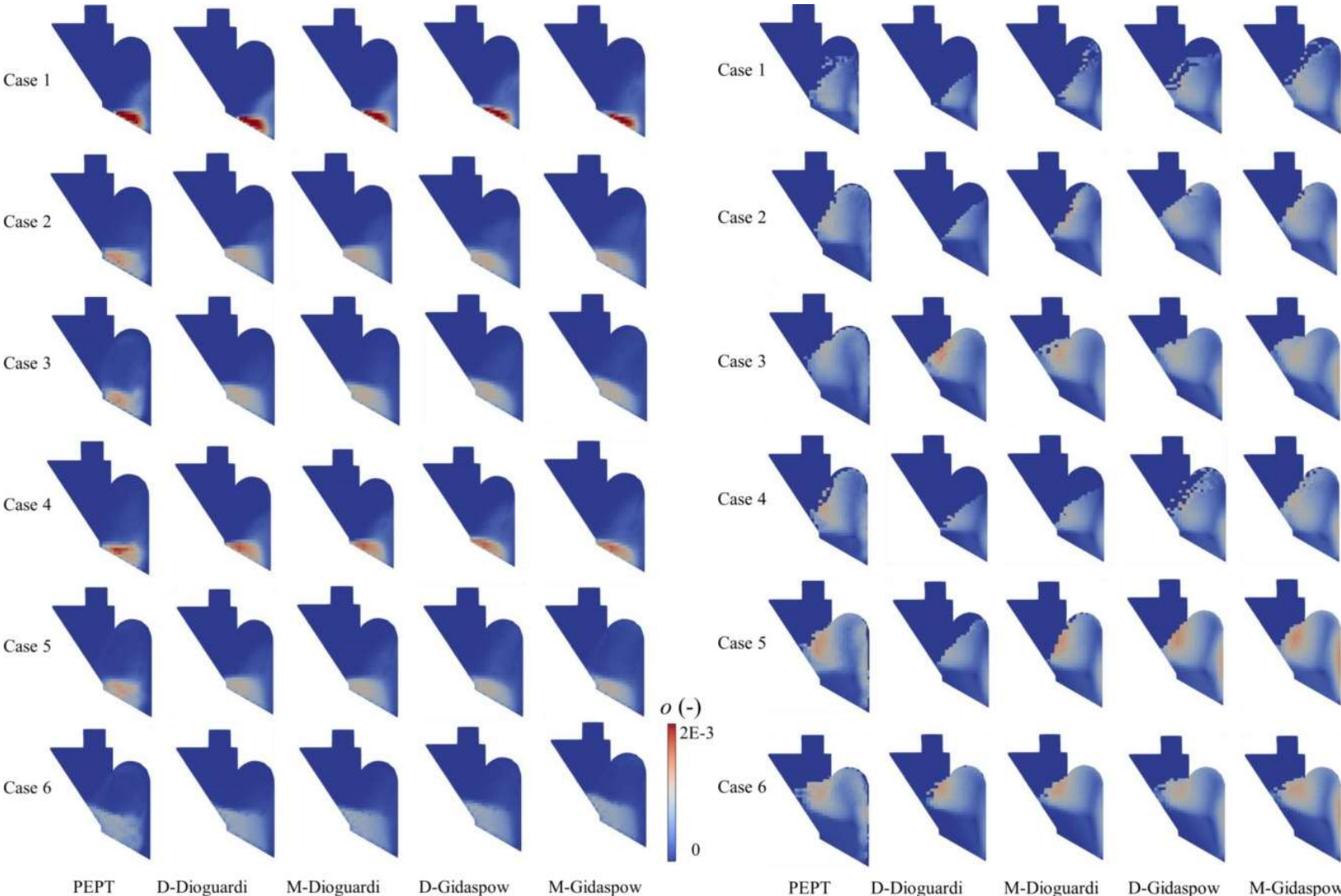
CFD-DEM  
Model 4

- Simulation accuracy can be **quantitatively assessed** through Pearson coefficient (or others statistical measures)
- Pearson coefficient can also be used as a **cost function** for ACCES calibration
- **Skip the characterisation step!**

	Model 1	Model 2	Model 3	Model 4
Occupancy	0.659	0.733	0.867	0.834
Velocity	0.942	0.930	0.913	0.903

# Rigorous, Quantitative Validation

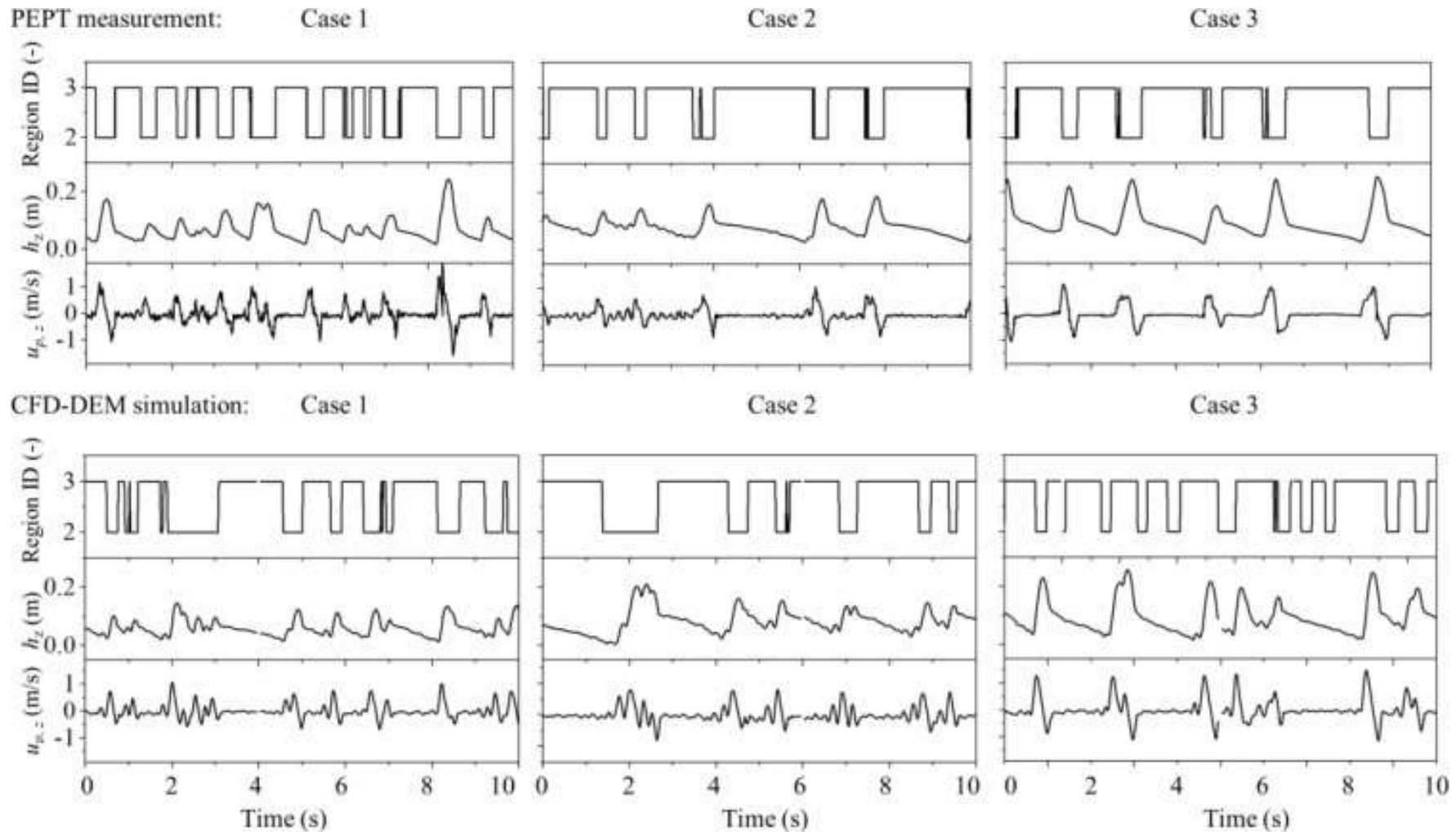
Che, **Windows-Yule**, et al. (2023). PEPT validated CFD-DEM model of aspherical particle motion in a spouted bed. *Chemical Engineering Journal*, 453, 139689.



For further rigour, can validate under **numerous system conditions** spanning full parameter space of interest...

# Rigorous, Quantitative Validation

Che, **Windows-Yule**, et al. (2023). PEPT validated CFD-DEM model of aspherical particle motion in a spouted bed. *Chemical Engineering Journal*, 453, 139689.

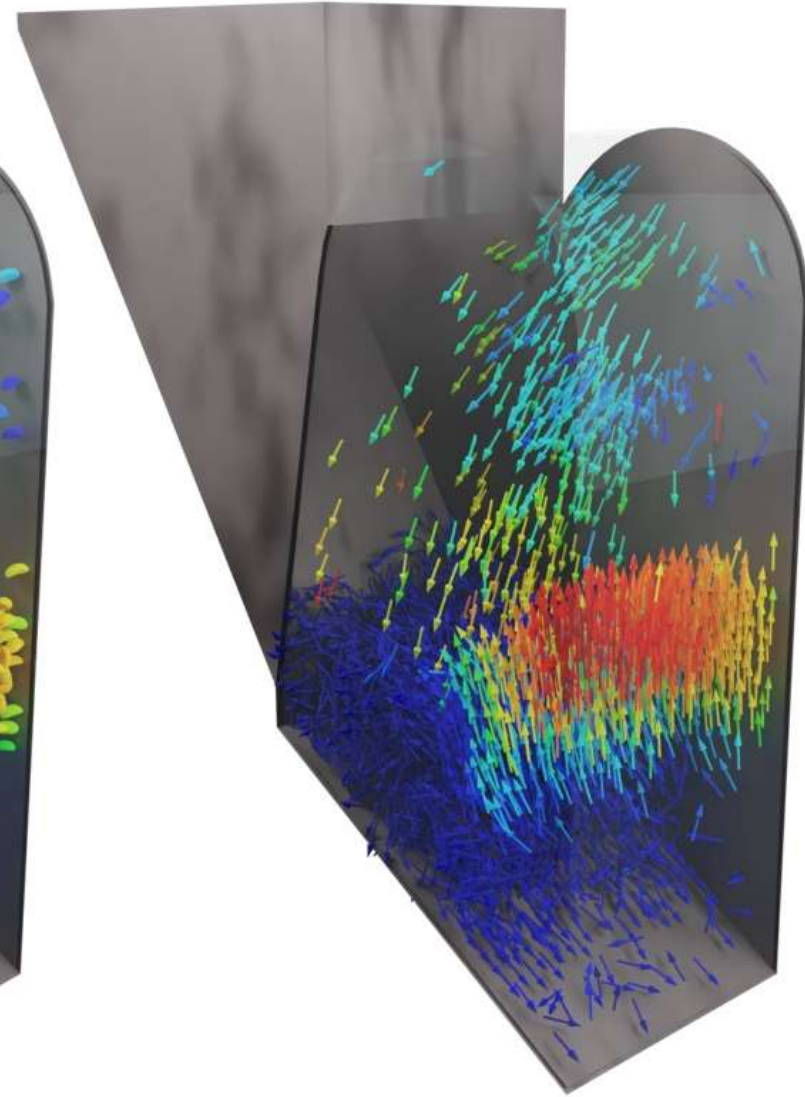
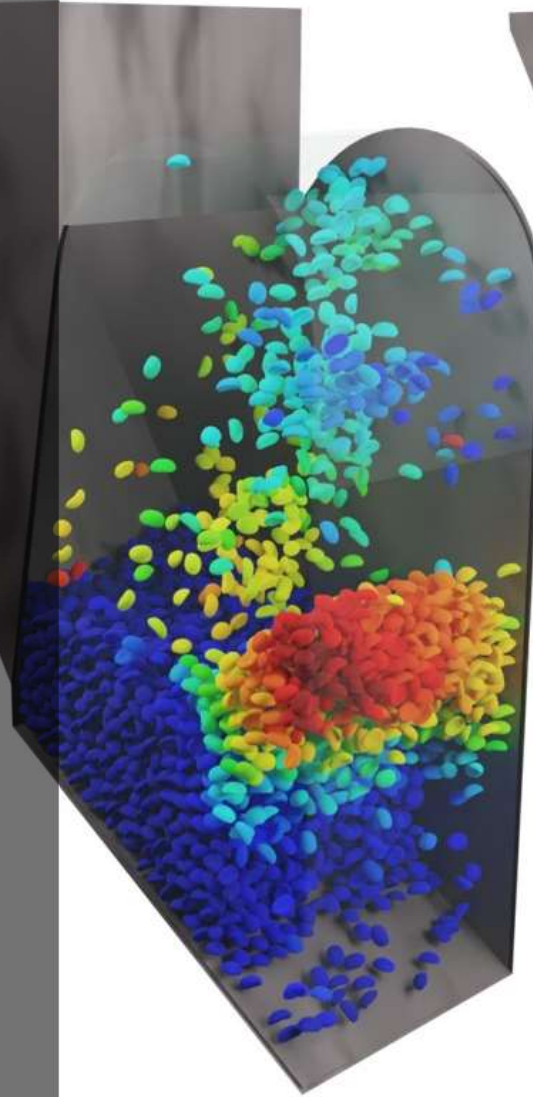


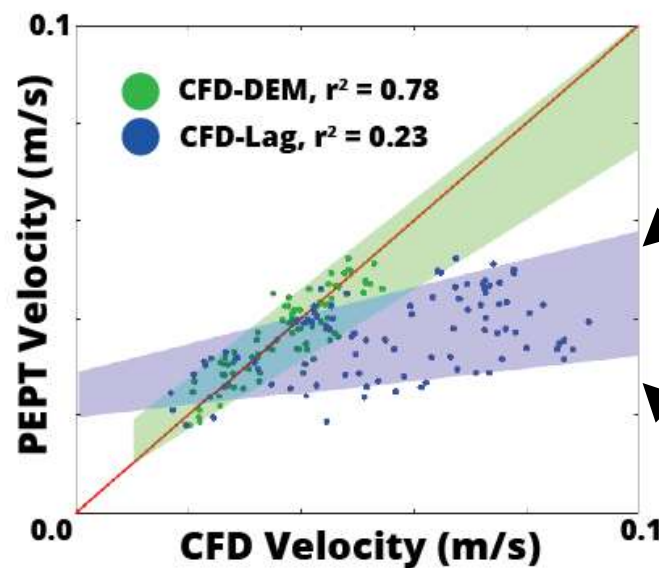
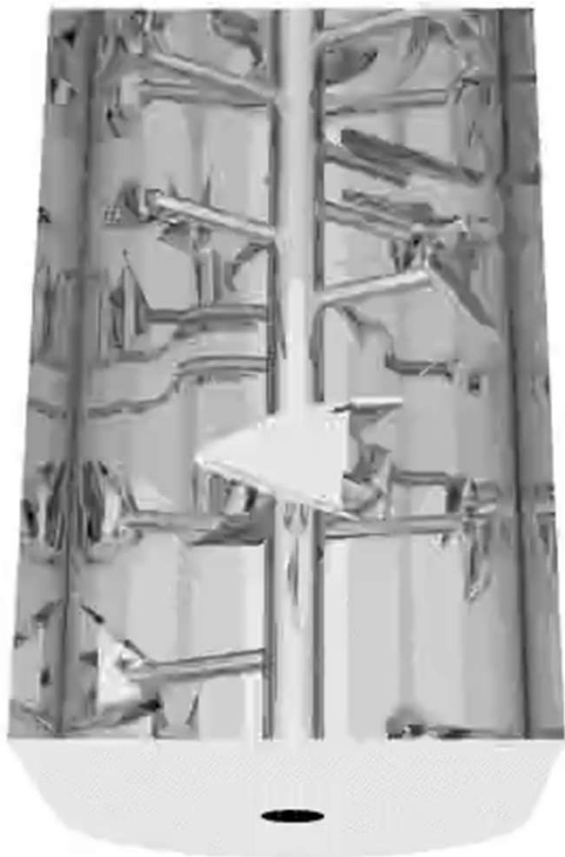
...as well as investigating a diverse array of other (**Eulerian & Lagrangian**) quantities!



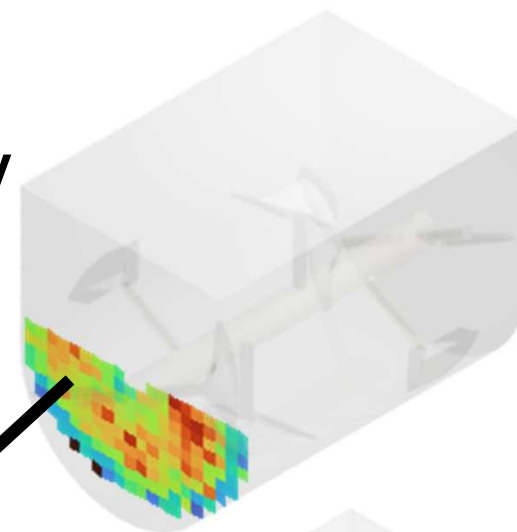


End result: a **comprehensively validated** numerical model which can be used to **easily, efficiently and cheaply** gain insight into JDE's systems.

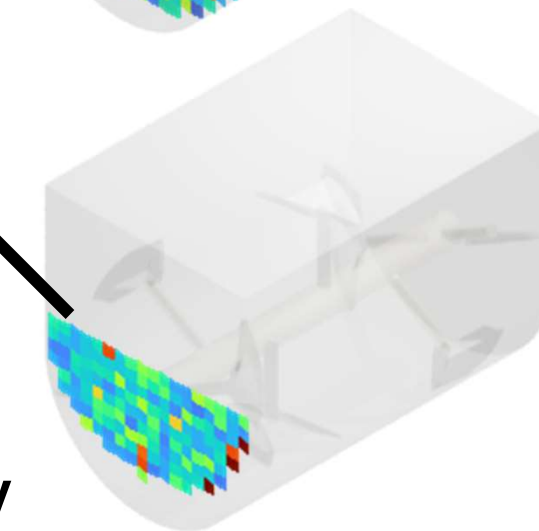




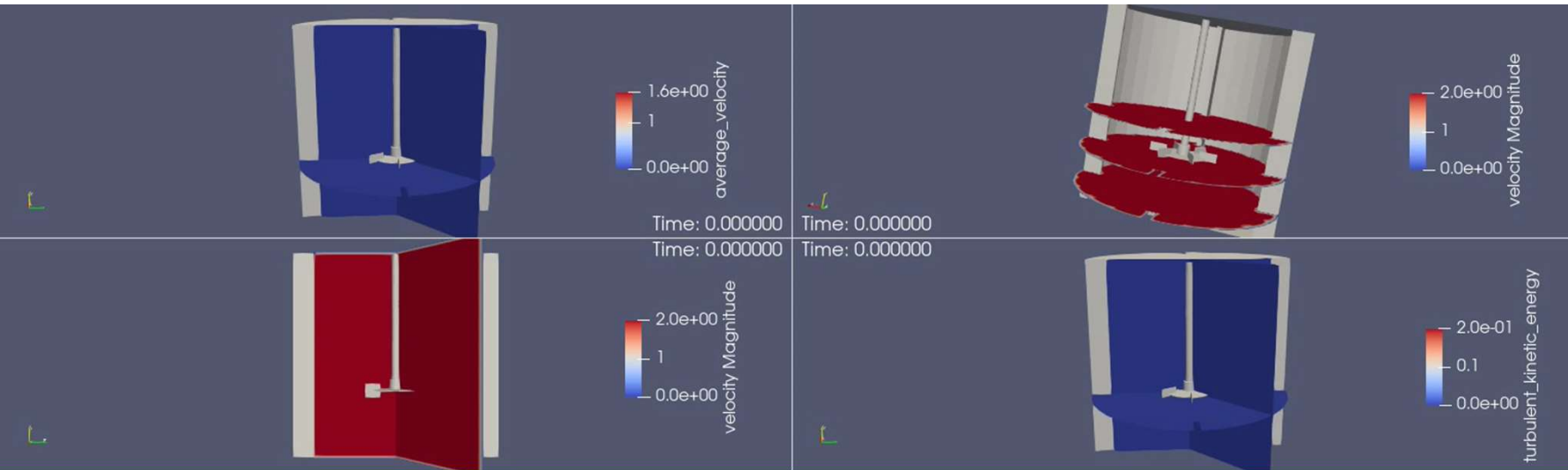
CFD  
velocity  
field



PEPT  
velocity  
field



Not *just* spouted beds, not '*just*' particles

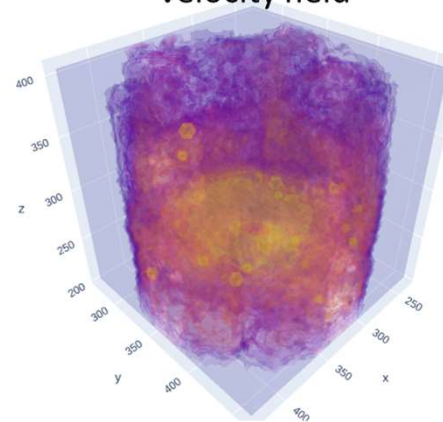


**JM** Johnson Matthey  
Inspiring science, enhancing life

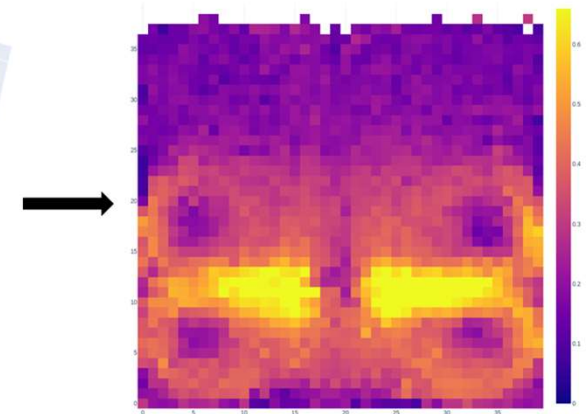
**ifp** Energies  
nouvelles

Not *just*  
spouted beds,  
not ‘*just*’  
particles

Full three-dimensional  
velocity field



Individual 2D “slice”





**MAPP**

HENRY · · · ·  
ROYCE · · · ·  
INSTITUTE



Royal Academy  
of Engineering

THE  
ROYAL  
SOCIETY

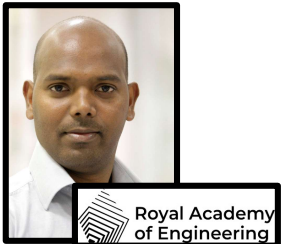


Engineering and  
Physical Sciences  
Research Council

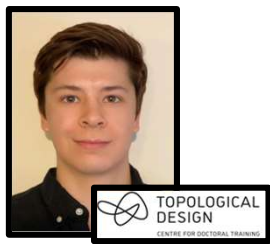
Funder  
Acknowledgements

Innovate UK

# The Team



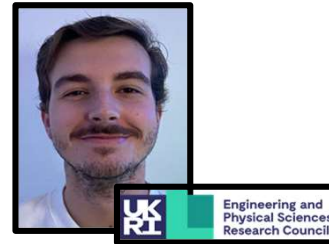
William Peace



Jack Sykes



Zoe Chu



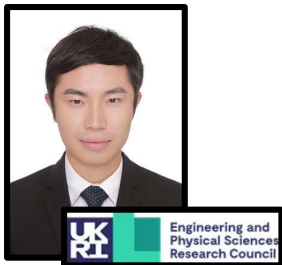
Matthew Herald



Khizra Abdul Wadood



Swapna Kudal



Ben Jenkins



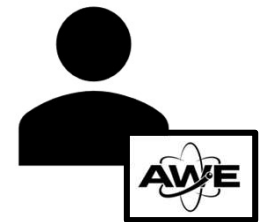
Owen Jones-Salkey



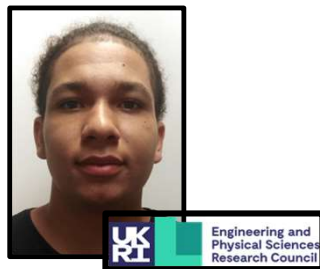
Leonard Nicușan



Dan Rhymer



Roberto Hart-Villamil



Niklas Adio



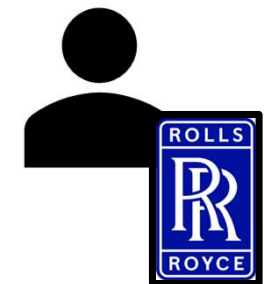
Dominik Werner



Issa Munnu



Dan Weston





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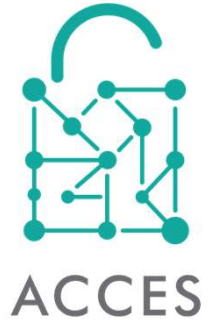


## IV. PEPT as a *Calibration Tool*



ACCES

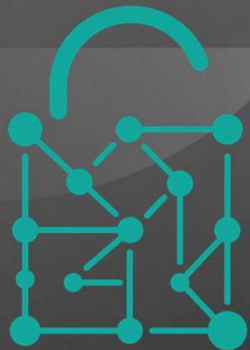
# Autonomous Calibration using Evolutionary Algorithms



- Calibration is an **infamously slow and difficult task**  
And one too often overlooked in the literature!
- As well as simply *validating* existing algorithms, PEPT can be used to *calibrate* simulations
- Specifically, it can be used to provide detailed objectives for **evolutionary algorithms**



**ACCES:**  
Autonomous  
Characterisation &  
Calibration using  
Evolutionary  
Simulation



ACCES

- I. Choose experimental system to model
- II. Define a cost function to quantify difference between experiment & simulation
- III. Choose a suitable optimiser
- IV. Set goal to minimise error function (i.e. maximise agreement between simulation and experiment)
- v. Iterate towards minimum (i.e. find 'true' DEM parameters)



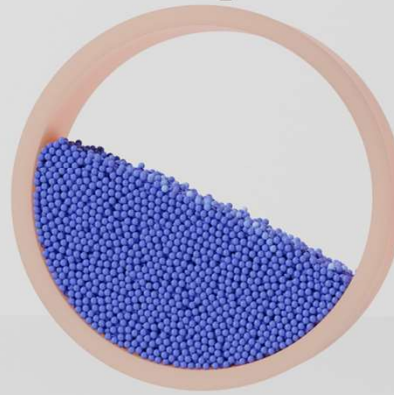
# I. Choosing a system

To illustrate ACCES in an accessible manner, let us consider a simple system to model: Granutools *GranuDrum*

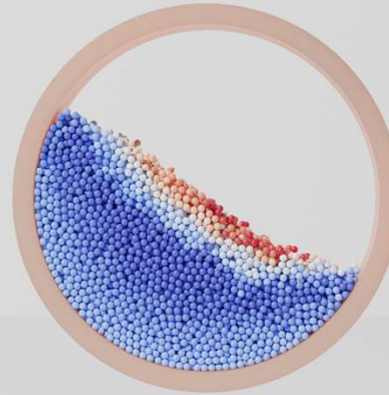
- Simple
- Industrially relevant
- Diverse phenomenology



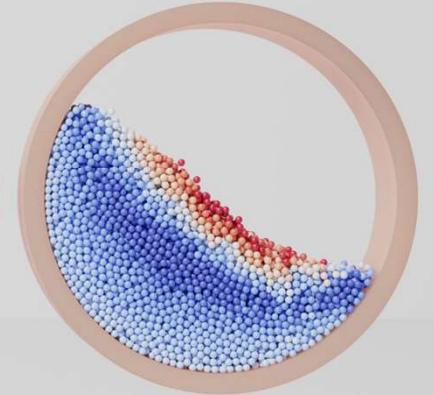
5 rpm



45 rpm

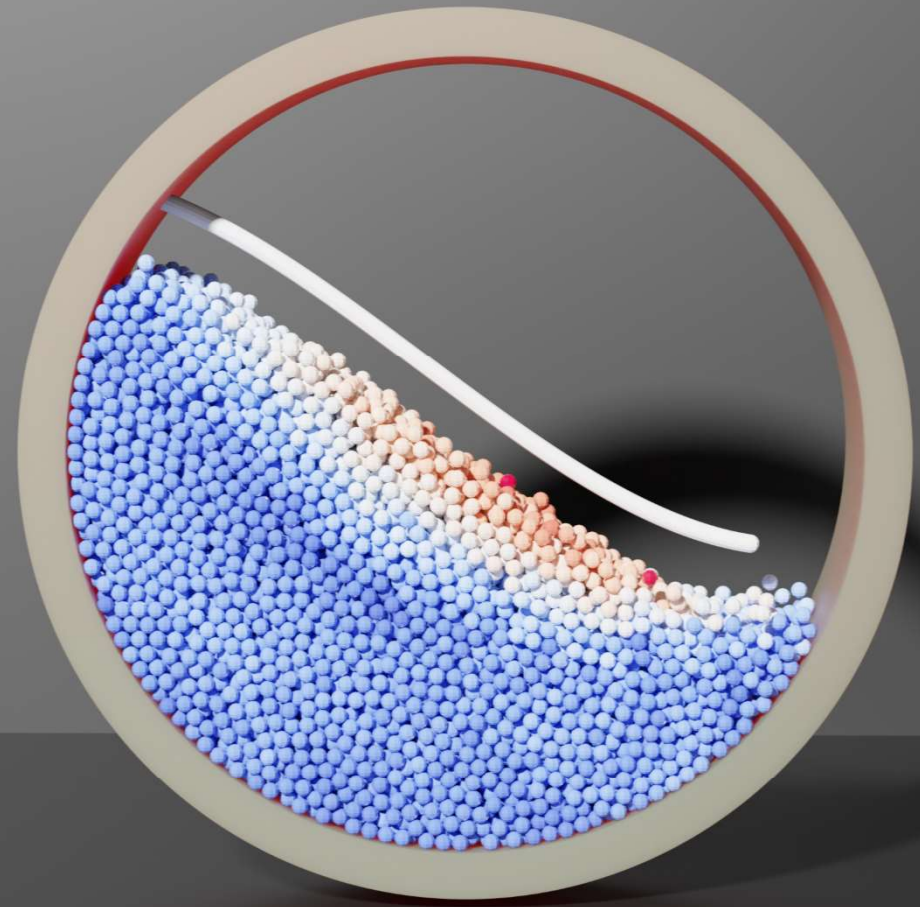


60 rpm



## II. Defining a cost function

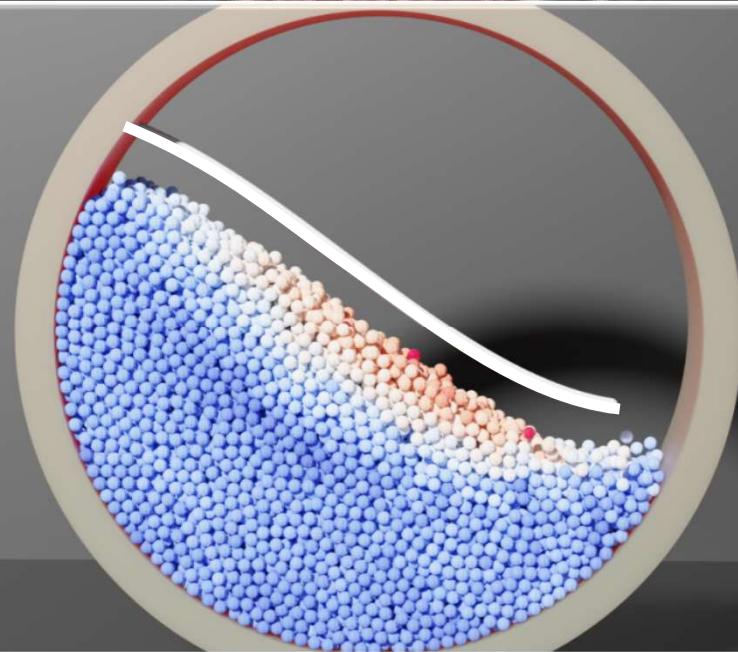
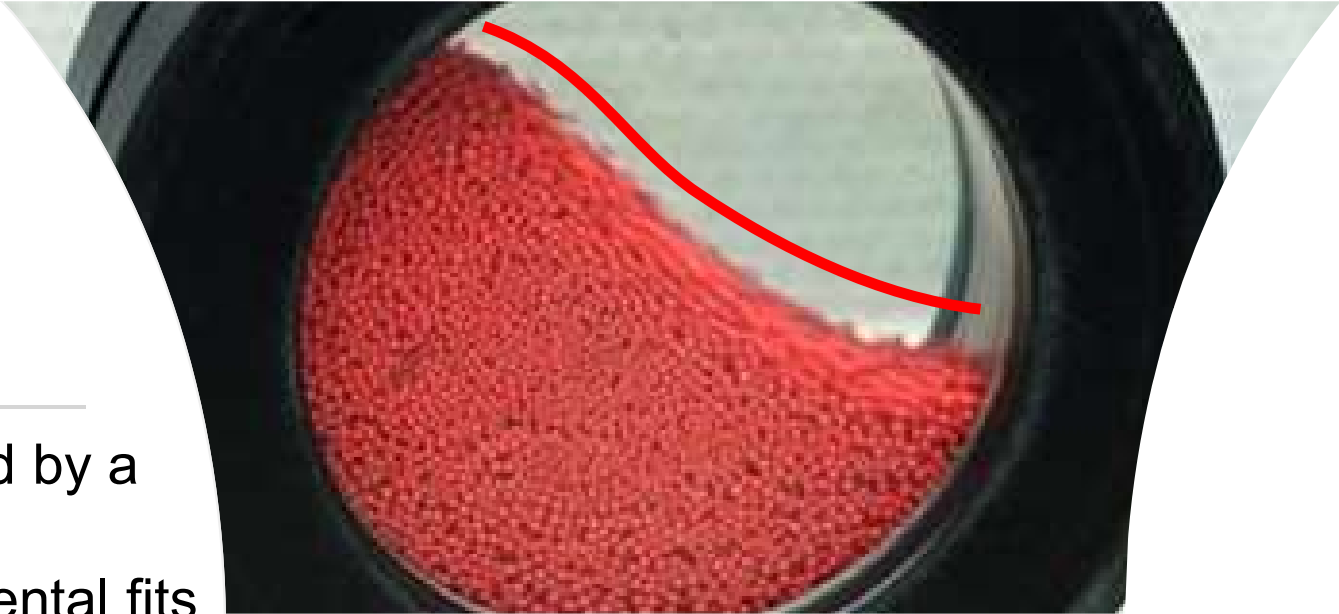
- Cost function can relate to **practically any quantity**
  - Mean system velocity
  - Velocity distribution
  - Density distribution
  - Granular temperature...
- Precise choice depends on goals of calibration
- In this example we want to obtain values for **sliding & rolling friction**
- → Free surface shape as cost function



$$f(\mathbf{x}) = 0.005 \mathbf{x}^3 + 0.002 \mathbf{x}^2 - 0.023 \mathbf{x} + 0.04$$

## II. Defining an error function

- Free surface can be characterised by a 3<sup>rd</sup> order polynomial
- Compare simulation and experimental fits

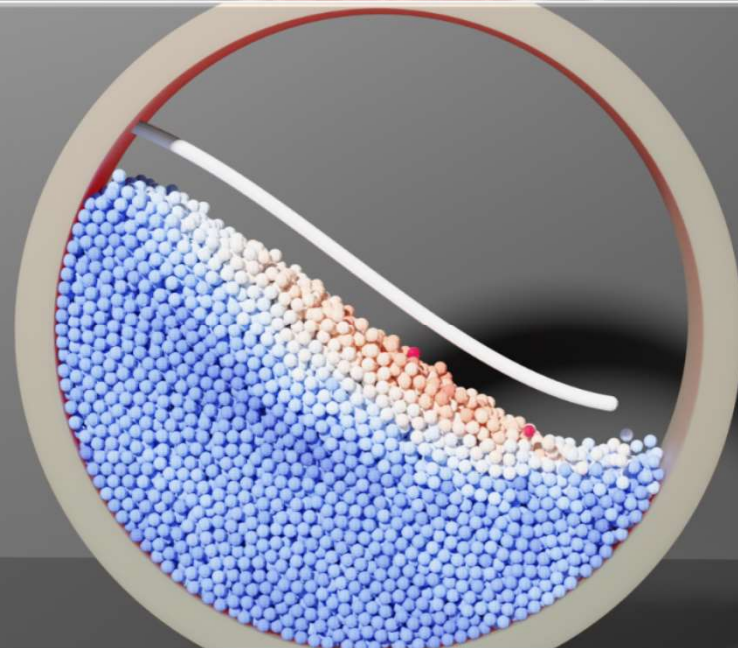
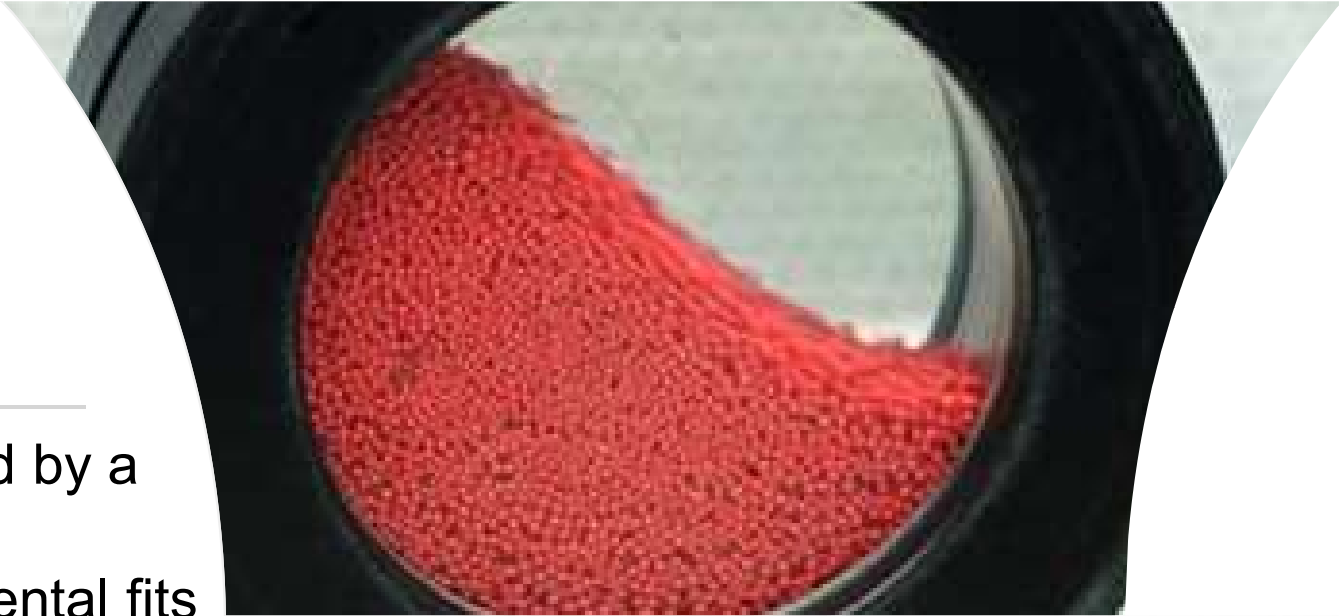
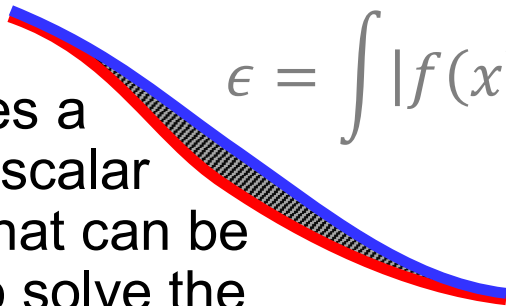


## II. Defining an error function

- Free surface can be characterised by a 3<sup>rd</sup> order polynomial
- Compare simulation and experimental fits
- Cost function taken as the integral of the absolute difference between the 2 polynomials

Provides a simple scalar value that can be used to solve the optimisation problem

$$\epsilon = \int |f(x) - g(x)| dx$$

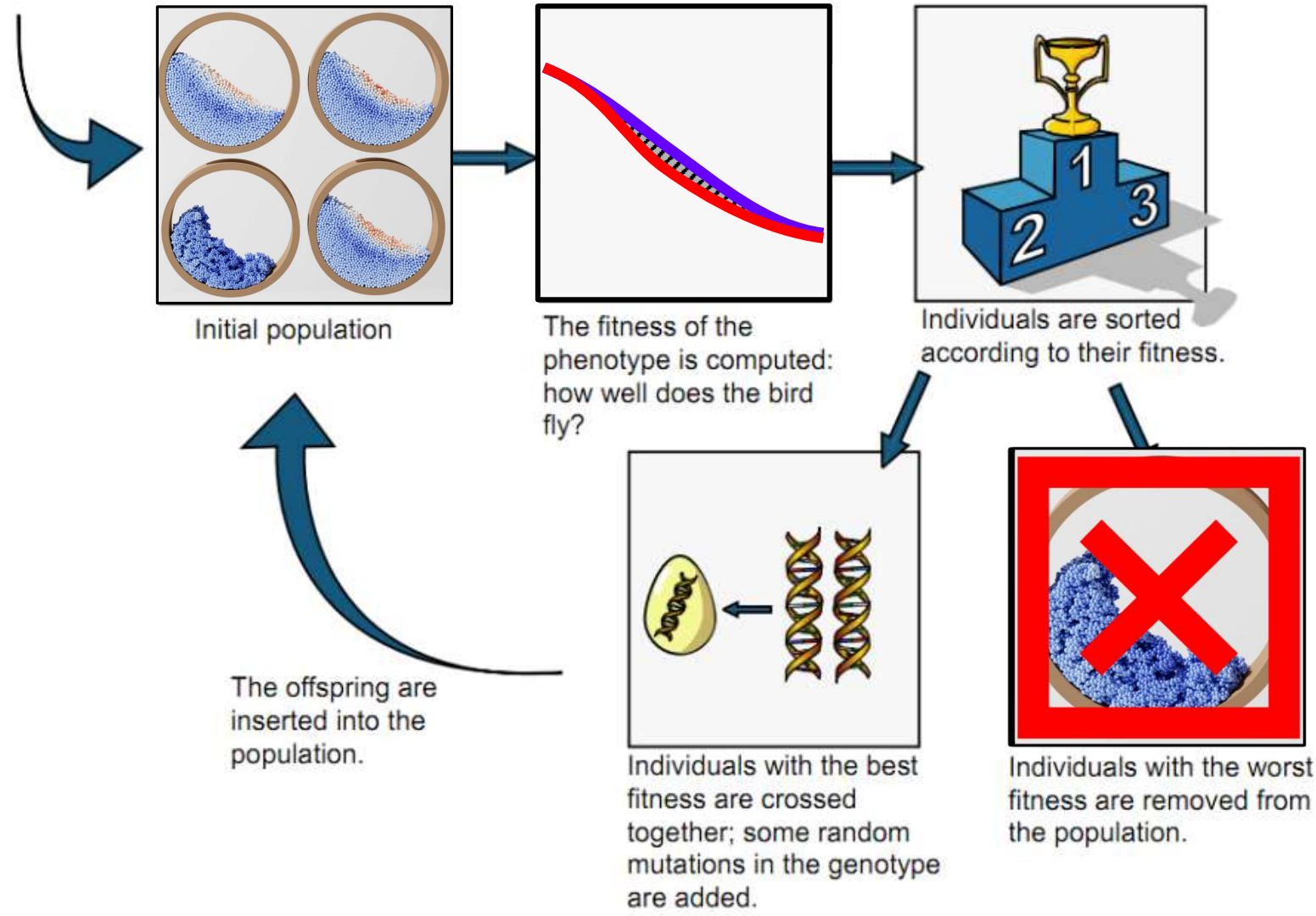


# Evolutionary Optimisation – How it Works

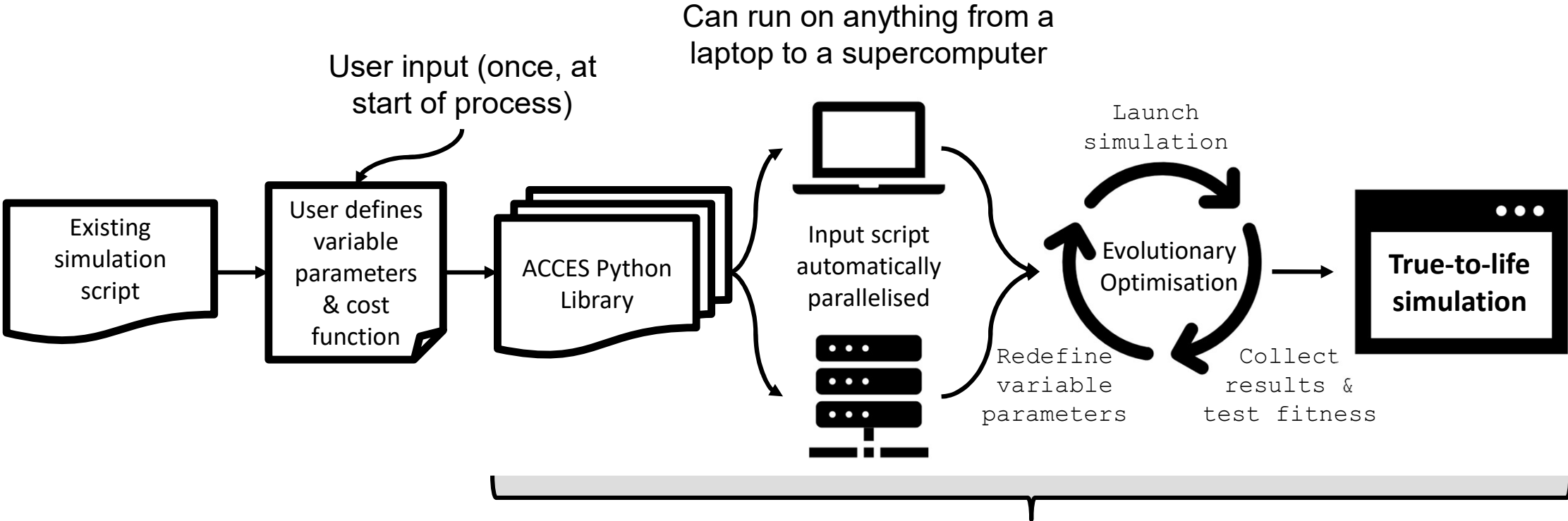
The problem with evolutionary algorithms: **lots of function evaluations**

Especially problematic when coupled to DEM

Utilise state-of-art **CMA-ES** algorithm, which adaptively changes as the spread of the function increases



# Evolutionary Optimisation – How it Works in ACCES



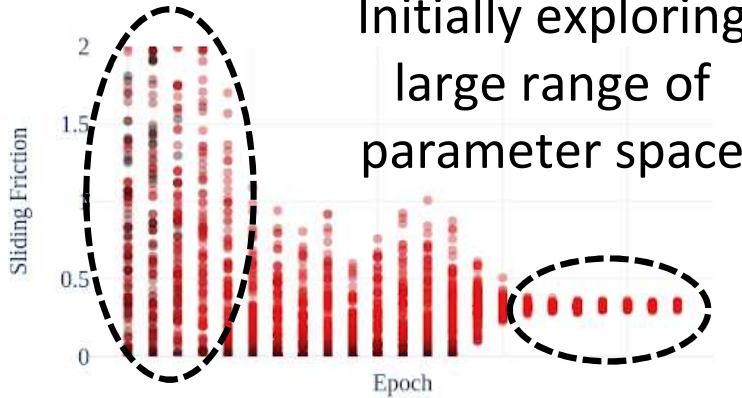
**Absolutely no user input required!**



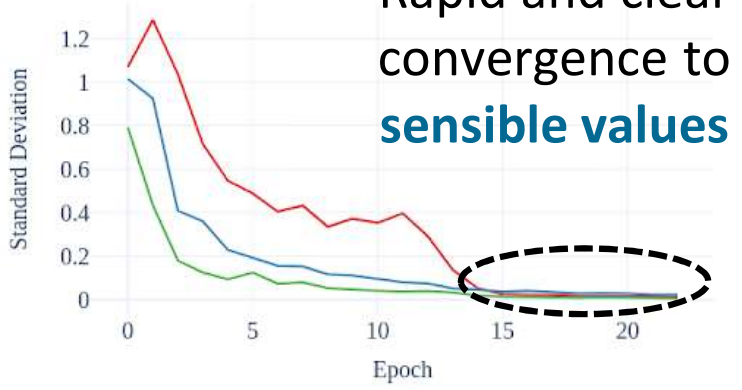
**Metaprogramming** (code that writes code): ACCES takes input scripts, understands them, hacks them, and modifies them to run in fault-tolerant massively parallel environments

# V. Iterating toward optimal parameters

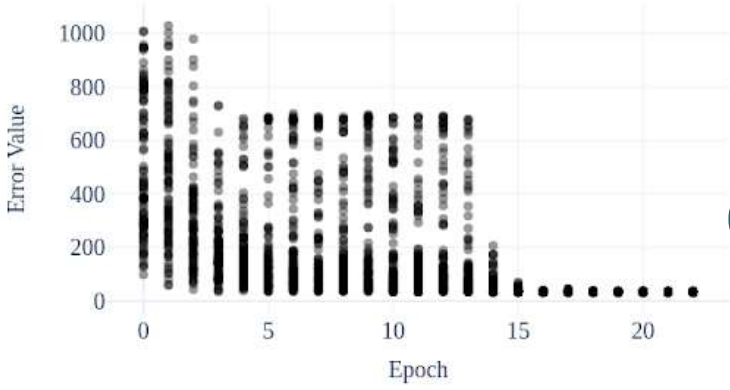
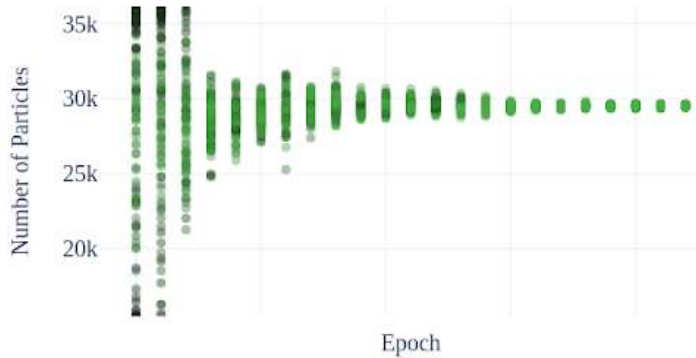
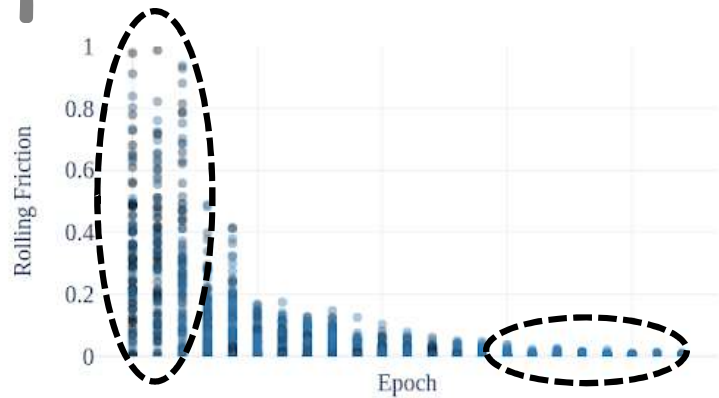
Initially exploring large range of parameter space



Rapid and clear convergence to **sensible values**



- ACCES converges on the parameter values that minimise the error values (cost function)
- It evolves a “family” of solutions in epochs (x-axis) towards the fittest individuals.



“Family size” of 100 (100 simulations per “epoch”)

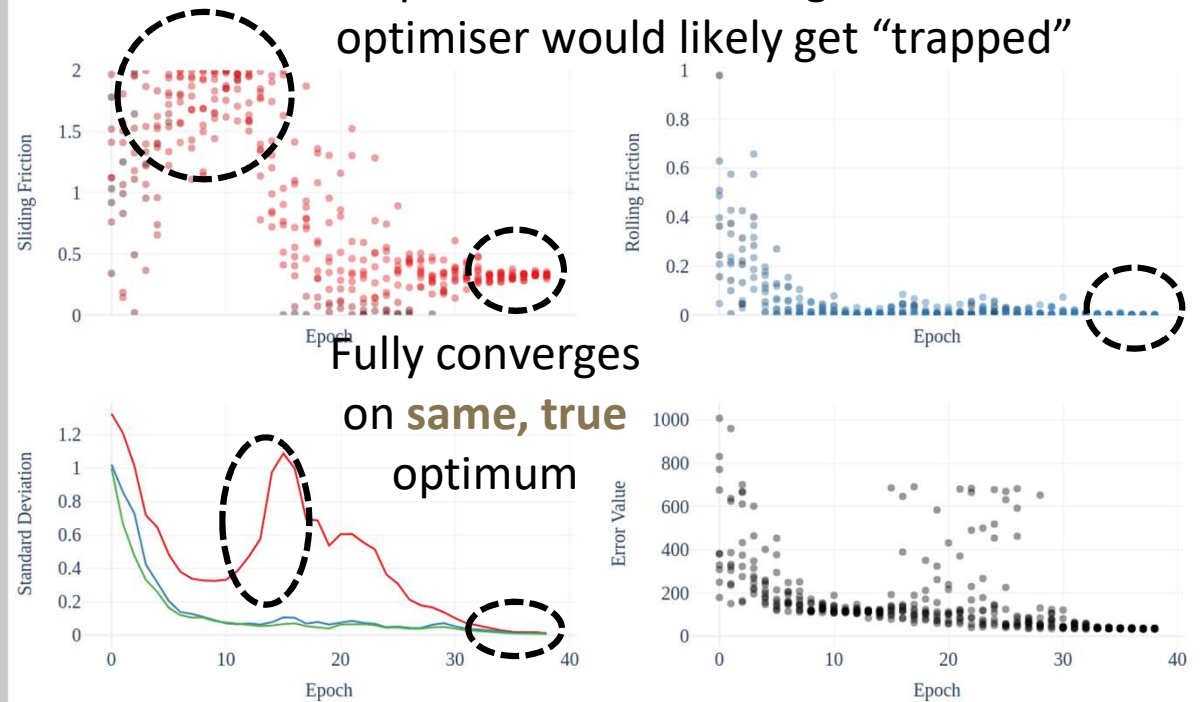
**Convergence in under 20 epochs!**

## V. Iterating toward optimal parameters

- What if I don't have 100 CPUs handy?
- “Family size” can be varied at will.
- Larger: more global search, fewer epochs
- Smaller: fewer simulations per epoch, more epochs
- → Fully scalable from HPC to Laptop
- **Can still autonomously “escape” false minima and reach true parameter values**

Same problem, **1/10<sup>th</sup> family size**

“Deep” local minimum – gradient-based optimiser would likely get “trapped”



Fully converges  
on **same, true**  
optimum

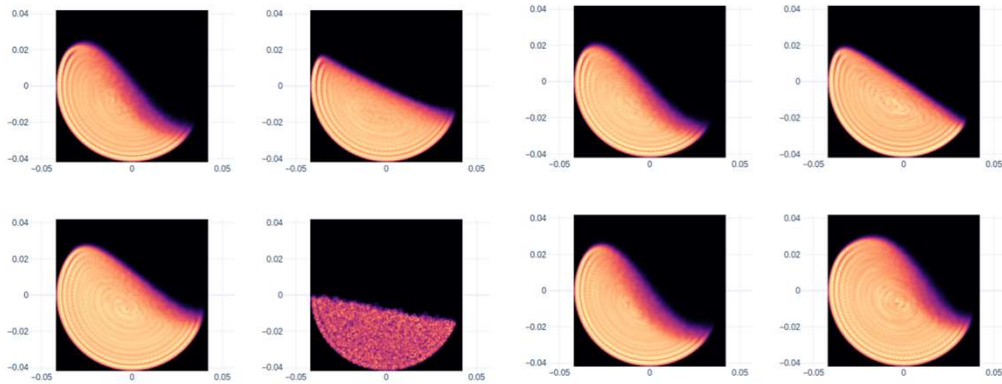
ACCES autonomously  
increases search area

Convergence in <40 epochs  
(<400 evaluations) with 2  
free parameters

C.f. ANNs & other optimisers for which 10,000 evaluations **per parameter** would be considered good!



## Early guesses



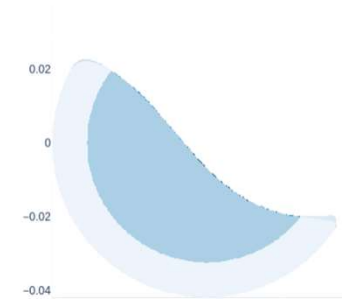
## Converged solution

Error of less  
than  $4 \text{ mm}^2$

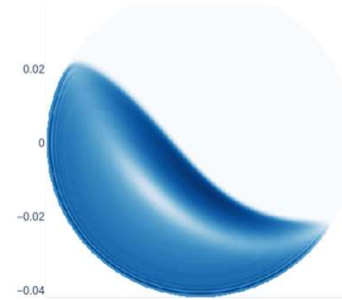
Experiment



Superimposed



Simulation



# Example solution

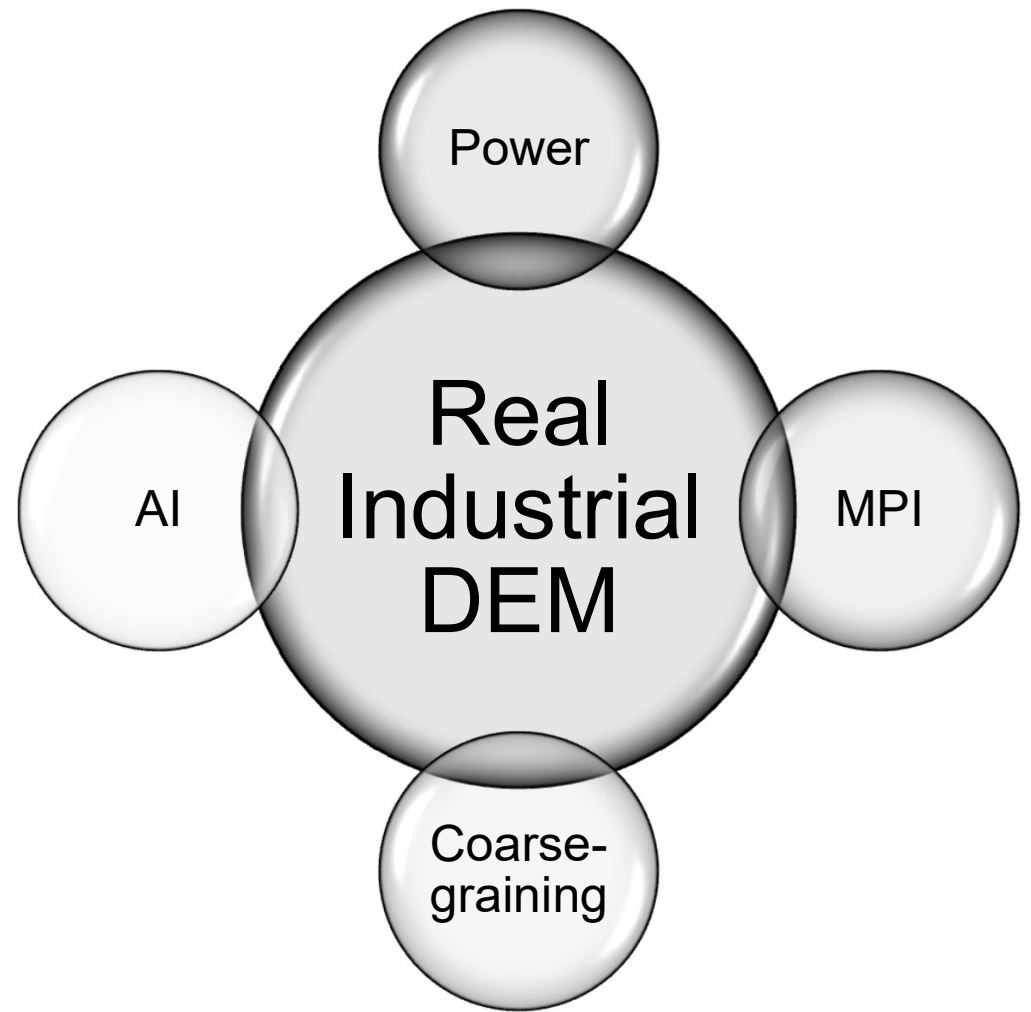
# Summary

- PEPT facilitates the detailed, 3D imaging of particulate, fluid, and multiphase media, even in large, opaque systems.
- Diverse array of quantities extracted from PEPT facilitates deep insight into the internal dynamics of both scientific and industrial systems
- Synergy between PEPT and numerical methods facilitates an optimal mix of *efficiency* and *accuracy* not achievable using either methodology in isolation

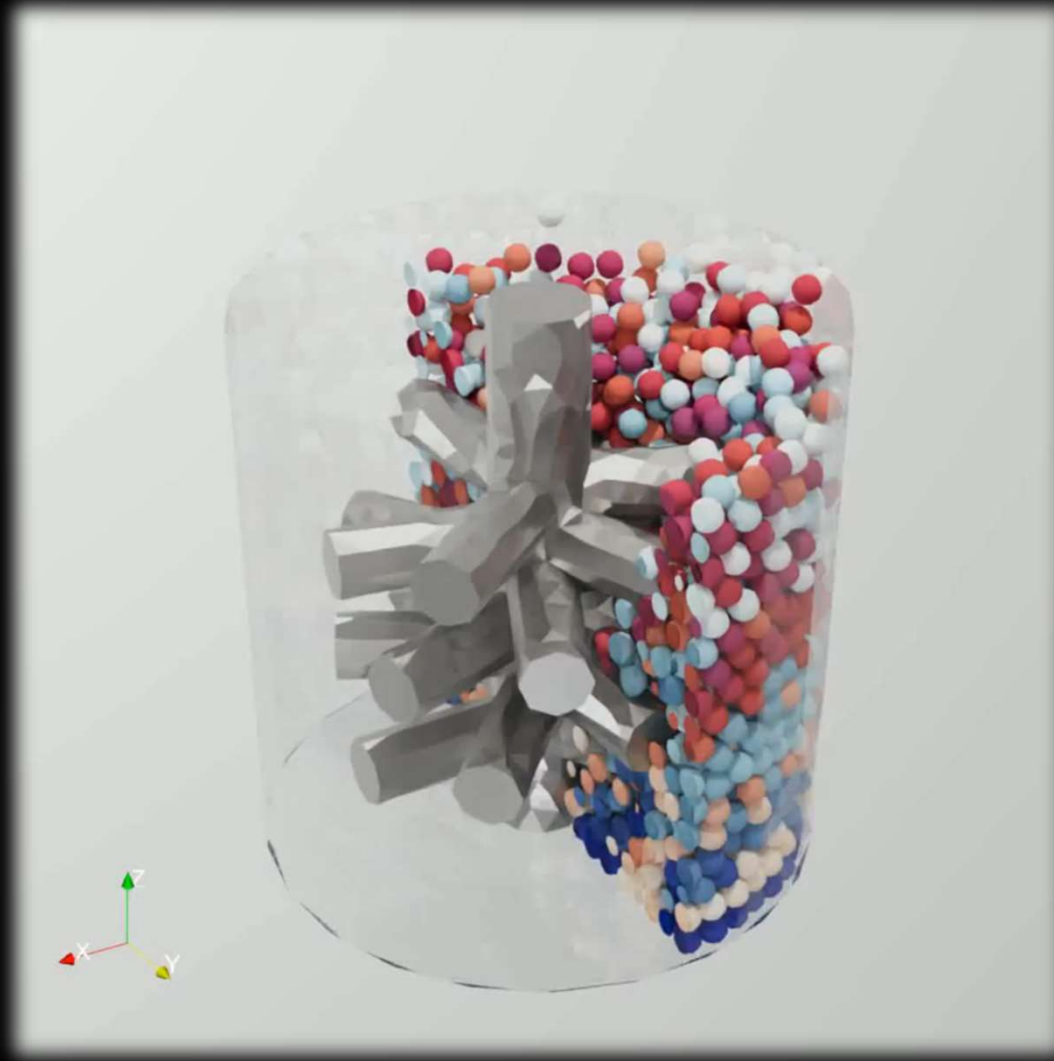


# Why is this talk timely?

- DEM is nothing new *but* recent advances in **computational power**, our ability to **parallelise** code, techniques for **upscaling** simulations, and advances in **AI** combined mean that today I can do things with DEM that at the start of my career I could only **dream of**
- Specifically, things like **this**:



# “Evolving” the optimal design for a unit operation





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# I. Evolutionary Algorithms as a Calibration Tool:

Autonomous Characterisation & Calibration using Evolutionary Simulation (**ACCES**)



ACCES

# Real-world motivation: the need for a better method of calibration



**IFPRI**

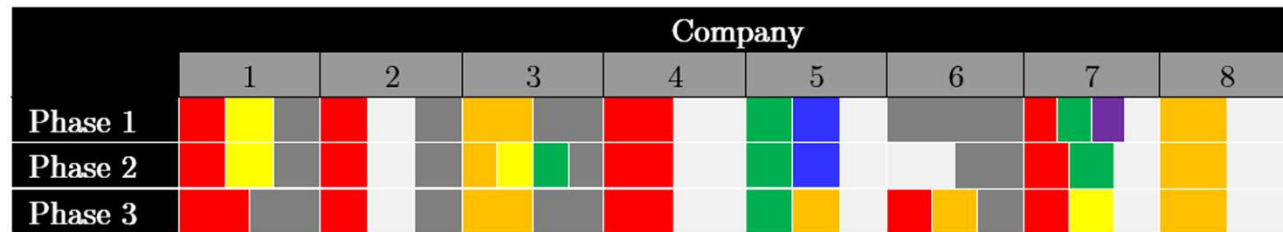
International Fine Particle Research Institute

- Leading a 5-year project with IFPRI to investigate current industrial DEM characterisation/calibration strategies
- Detailed interviews with 8 (now 12) multinational companies who use DEM, spanning Agriculture, Chemical, FMCG, Food, & Pharmaceutical sectors
- **Goal: to determine a “gold standard” methodology for the calibration of DEM simulations**



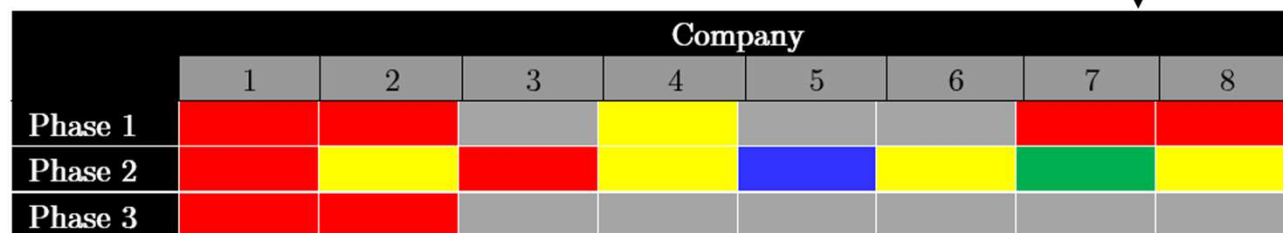
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# Real-world motivation: the need for a better method of calibration



Key: Characterisation methods	
Shear testing	Red
Angle of repose testing	Yellow
Impact testing	Green
Microtribometry	Blue
Ramp rolling friction testing	Purple
Laser diffraction	Grey
Microscopy / Optical imaging	Grey

Key: Shape model	
Rolling friction	Red
Glued sphere	Yellow
Superquadric	Green
Polyhedral	Blue
None	Grey



- **No two companies** adopted the same procedures, equipment, or geometric models
- Most produced **different values for same materials**
- The result?



# Real-world motivation: the need for a better method of calibration



**IFPRI**

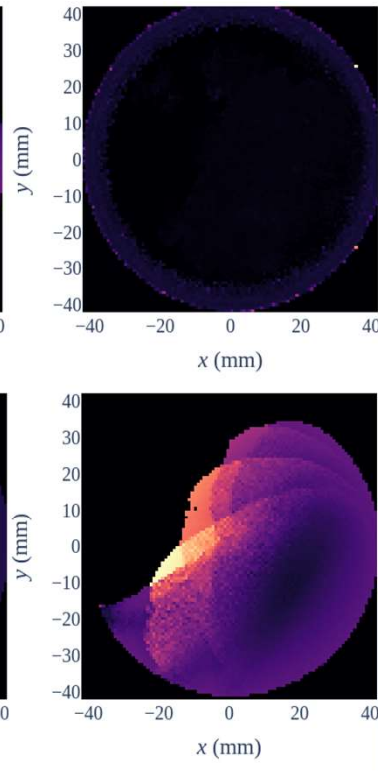
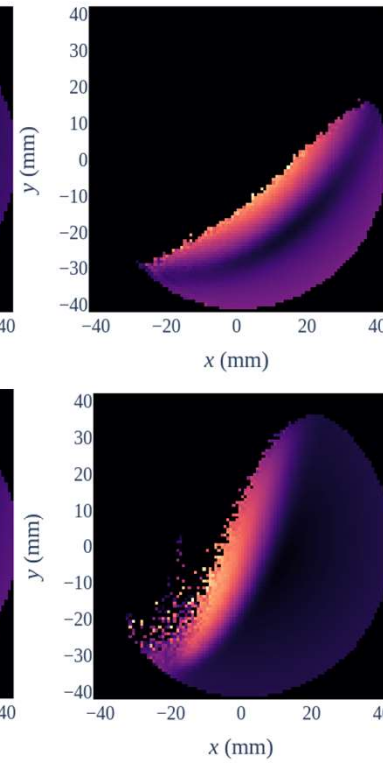
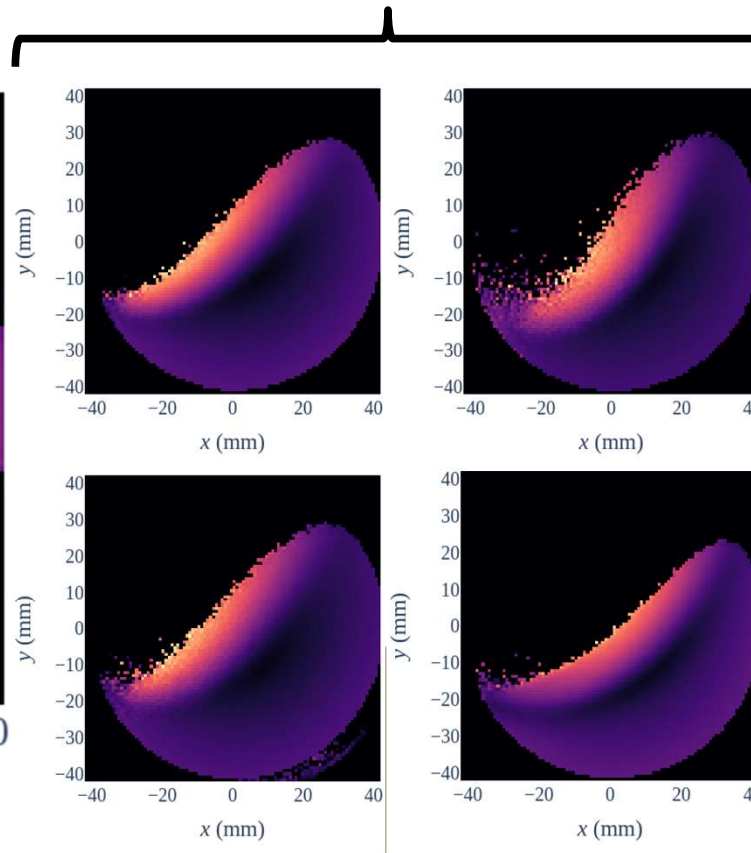
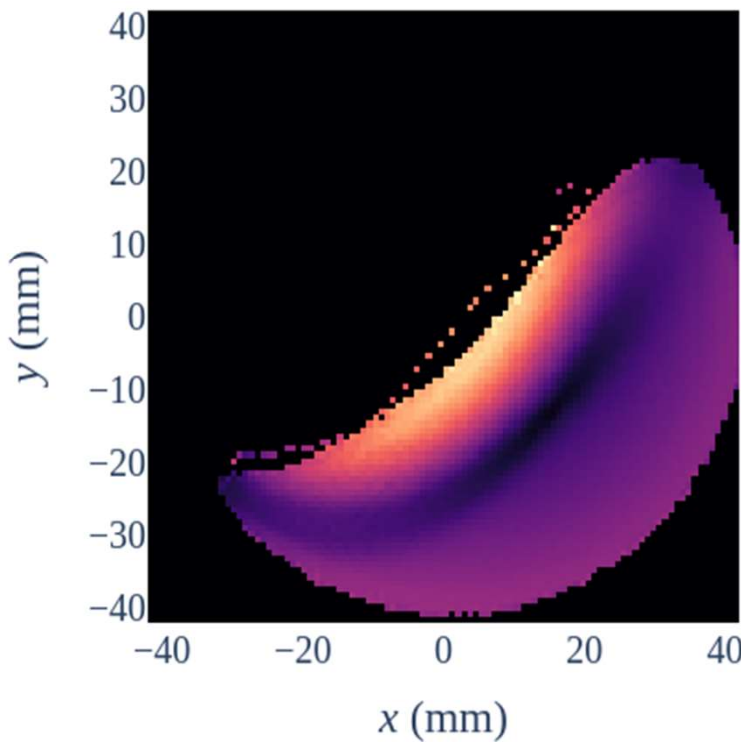
International Fine Particle Research Institute

Real, experimental  
PEPT data  
(more on this later)

Close, but no cigar

Not even  
close

Physically  
impossible!





# Real-world motivation: the need for a better method of calibration

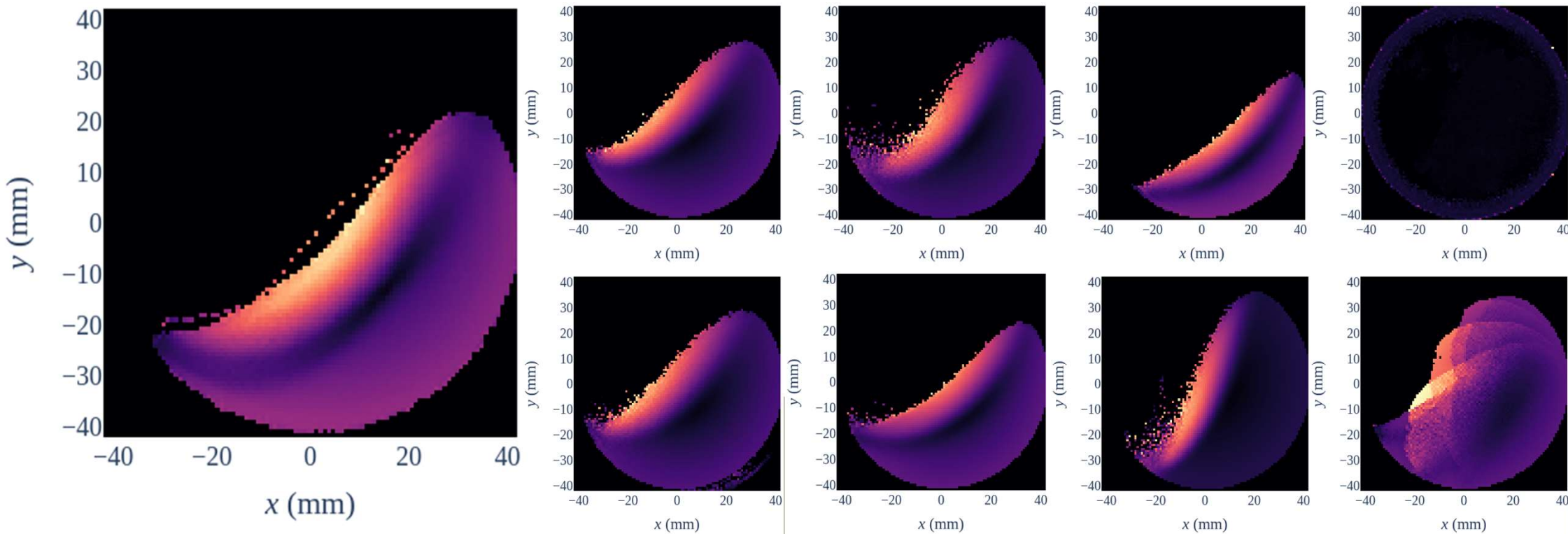


**IFPRI**

International Fine Particle Research Institute

**The scary part(s):**

1) These are *real methods* used by *real companies*



# Real-world motivation: the need for a better method of calibration

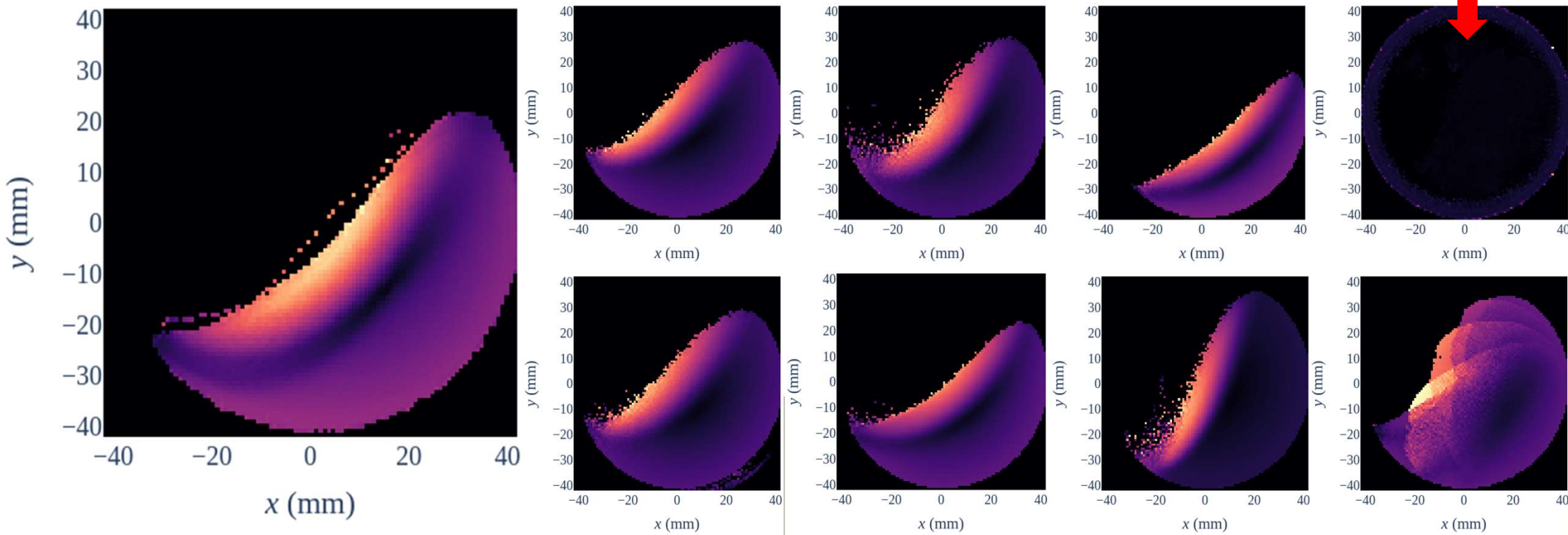


**IFPRI**

International Fine Particle Research Institute

**The scary part(s):**

2) While a sensible operator will re-try these...  
...without a technique like PEPT, how would we know the others are inaccurate?



# Real-world motivation: the need for a better method of calibration

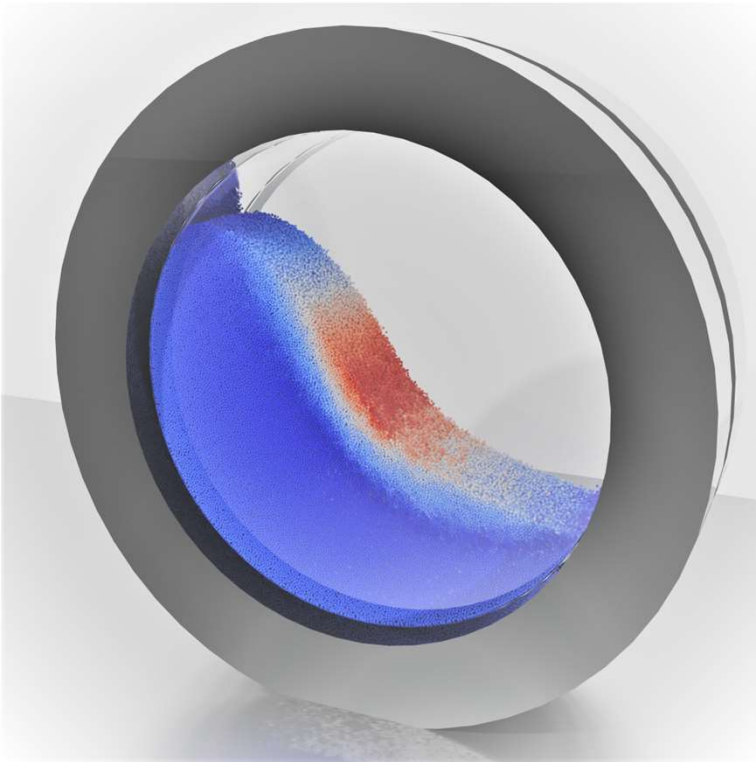


**IFPRI**

International Fine Particle Research Institute

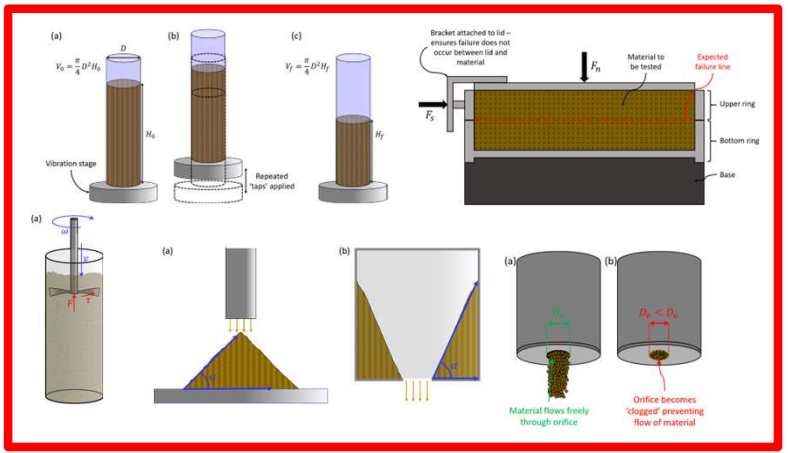
## The scary part(s):

3) This is for a relatively *simple, single-phase system* containing only *spherical particles*. How will these methods stand up for more complex cases?

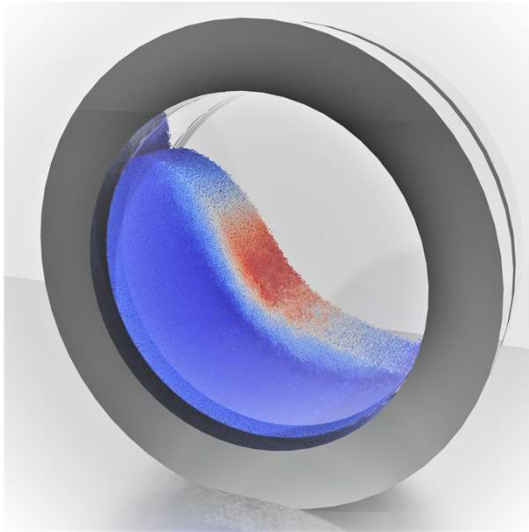


# Problem statement

- Particles' **bulk** properties are quick and easy to measure using easily-available equipment and standardised procedures.
- Measurement of particles' **microscopic** properties... is none of the above
- → We need a quick, easy and reliable way to map bulk measurements to microscopic properties



$\varphi_w$   
 $CI$   $\sigma_c$   $HR$   
 $\delta$   $AOR$   
 $\varphi$   $f$   $f_c$

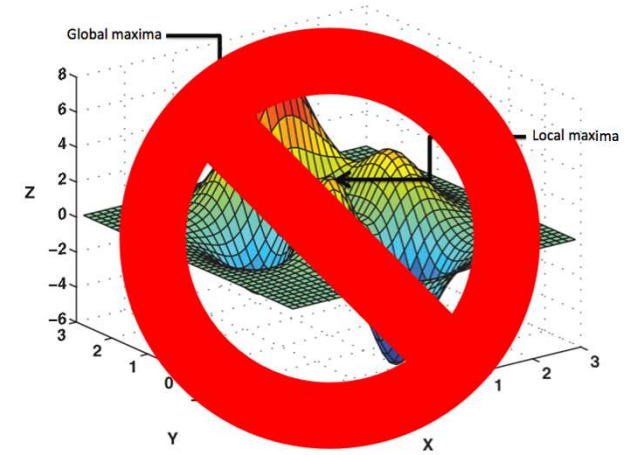


$\mu_s$   $\epsilon_n$   
 $\mu_t$   $\tau_c$   $\mu_r$   
 $E$   $CED$   
 $\epsilon_t$

# III. Choosing an Optimiser

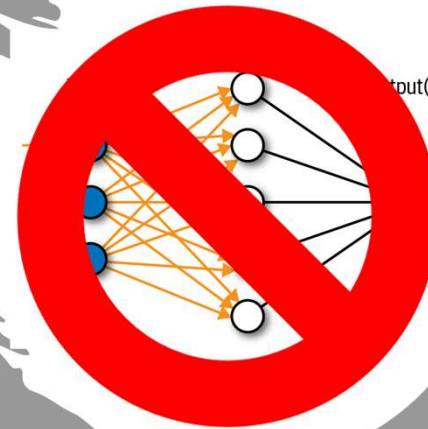
Error function can be **very non-convex** (local minima) and **very non-smooth** (lots of “jiggle”) → cannot trust gradient!

→ Evolutionary algorithms are the only logical choice for calibration-by-optimisation

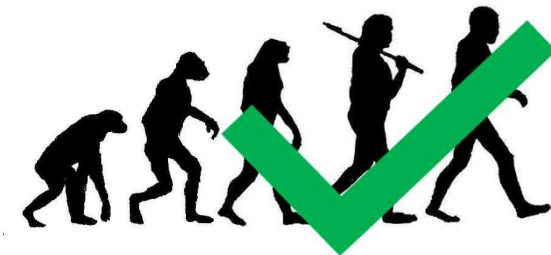


Gradient-based optimisers

ANNs

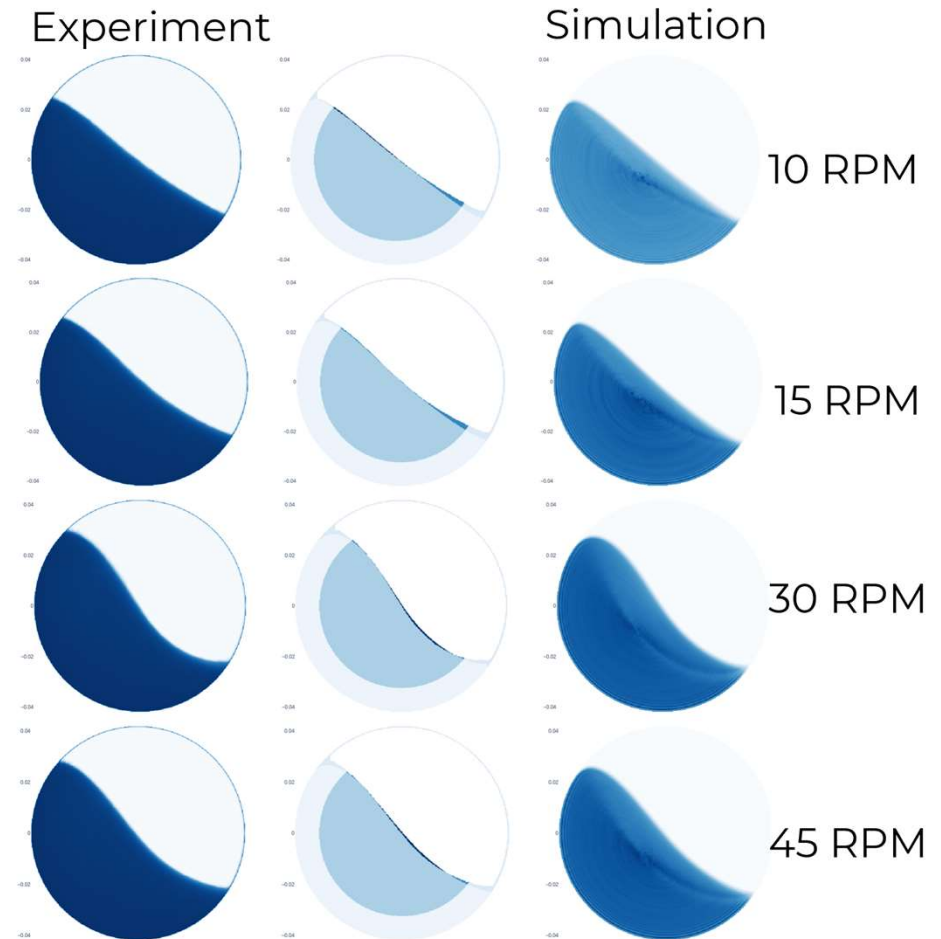


Evolutionary Algorithms



# Calibrating Multiple Parameters

- Mathematically, to solve for **N unknowns** we need **N closure relations**
- → Calibrating 5 parameters against a single measurement is **ill-defined**
- But this does not mean we need 5 instruments!
- E.g. a GranuDrum's free surface shape can be fitted by a 3rd order polynomial → 3 outputs!
- ACCES can calibrate against **multiple measurements** – e.g. GranuDrums at different RPMs, Shear Cells, FT4...
- → Drum can (hypothetically) calibrate  $3N$  parameters by running at  $N$  distinct RPM

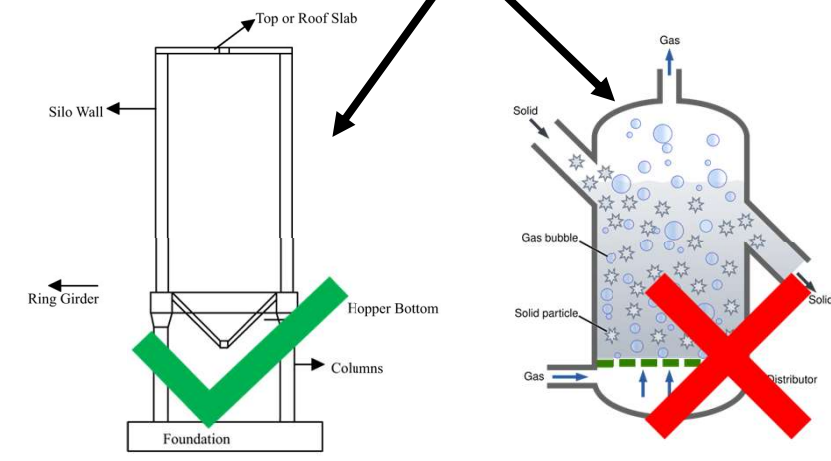
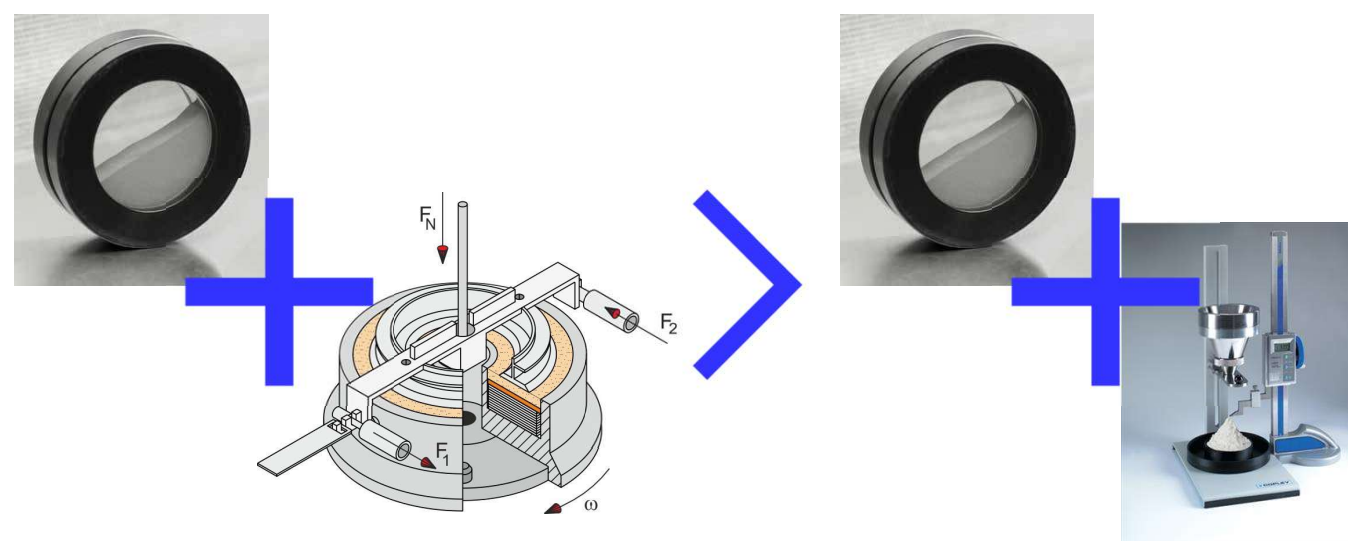
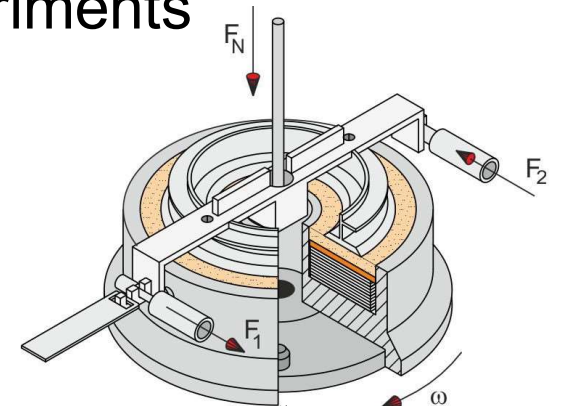


# Final thought on ACCES: A tool is only as good as its user

- Easy to make ACCES seem “too good to be true”
- In reality, though the **process** is fully automated, **human intelligence is still required** in the **initial design** of calibration experiments

ACCES can only work with what we give it!

- IFPRI project has highlighted importance of:
  - 1) Matching the calibration device to the “real” system
  - 2) When using multiple tools, choosing **distinct** tools





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## II. Evolutionary Algorithms as an Optimisation Tool:

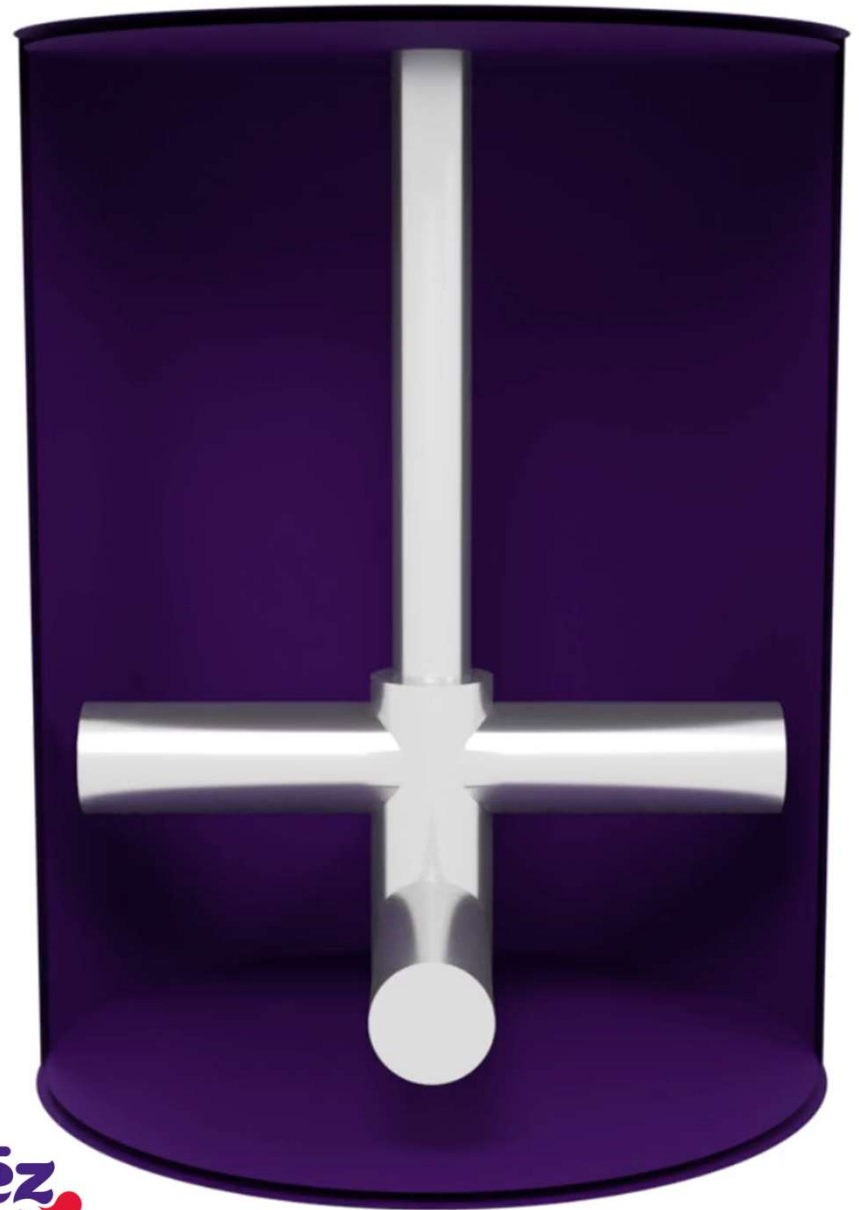


**Highly-Autonomous Rapid Prototyping for Particle-handling Processes (HARPPP)**



# Beyond calibration

- We have used ACCES to perfectly calibrate a simulation of (say) a mill
- So what next?
- For industry, typically:
  - Improve efficiency
  - Improve productivity
  - Reduce waste
  - → Improve green credentials
  - → Increase profit
- In other words, we have optimised **calibration**, now we want to optimise the **system itself**



# Optimising a Mill

Two main options:

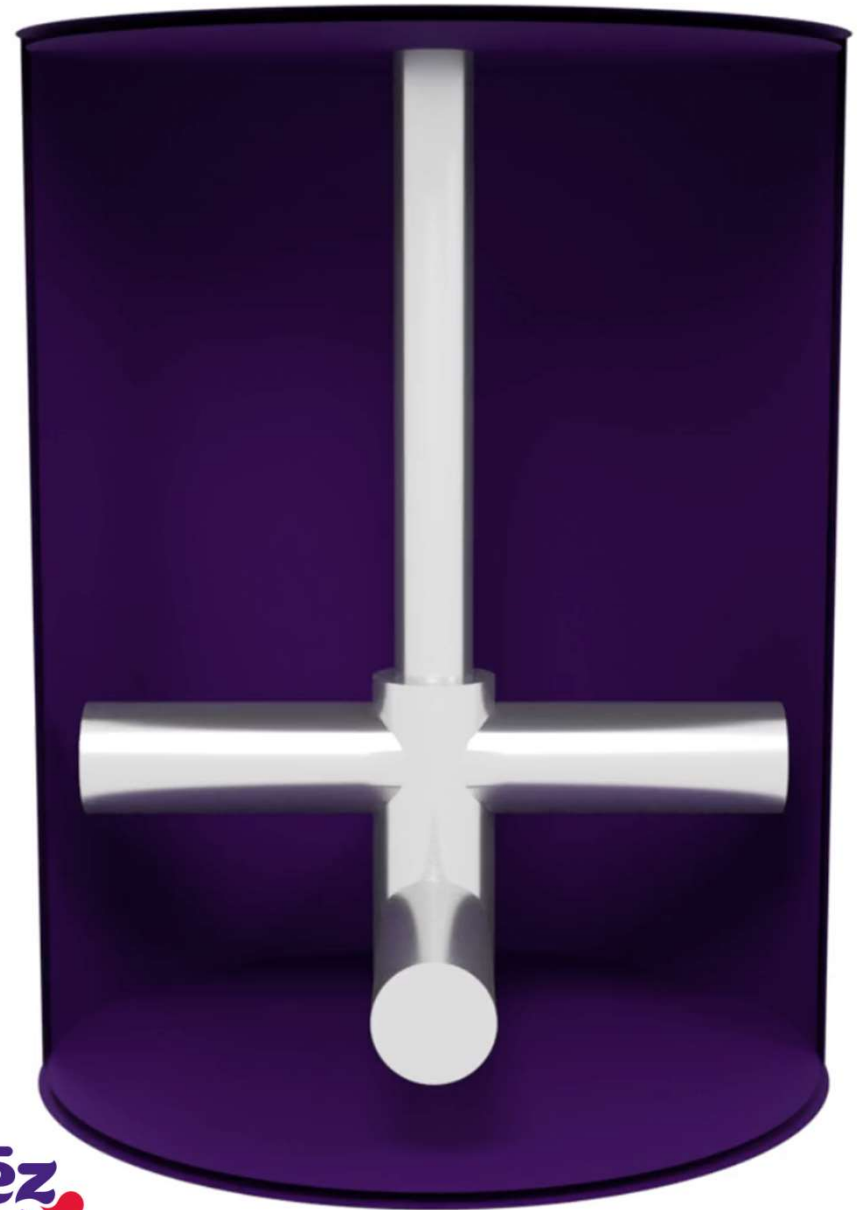
## 1. Optimise *process parameters* (e.g. attritor RPM, fill level...)

(Relatively) simple, easy to achieve both in “real life” and in simulation.

## 2. Optimise *geometry*

Highly costly in real life. Time-consuming, labour-intensive and “hit and miss” both in real life & DEM.

Can we 1) remove the element of chance and 2) remove the need for human input?

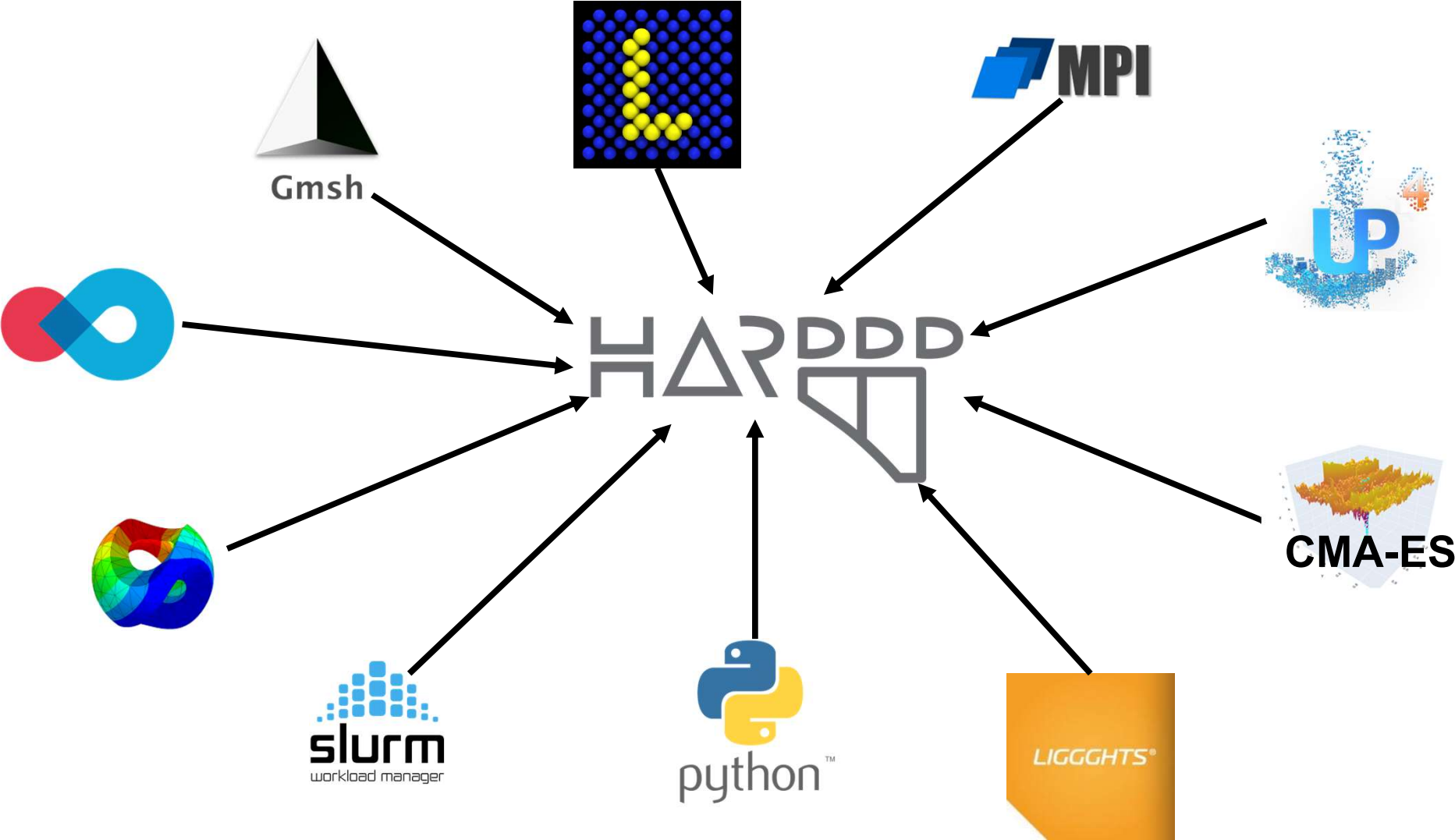


# Highly Autonomous Rapid Prototyping for Particulate Processes (HARPPP)

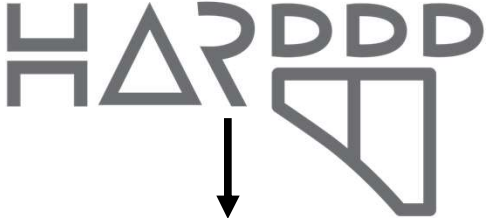


- Applying the evolutionary approach of ACCES to “real” optimisation
- Metaprogramming allows not only alteration of simulation scripts, but also **the autonomous design and implementation of entirely novel geometries**
- Not a simple task!

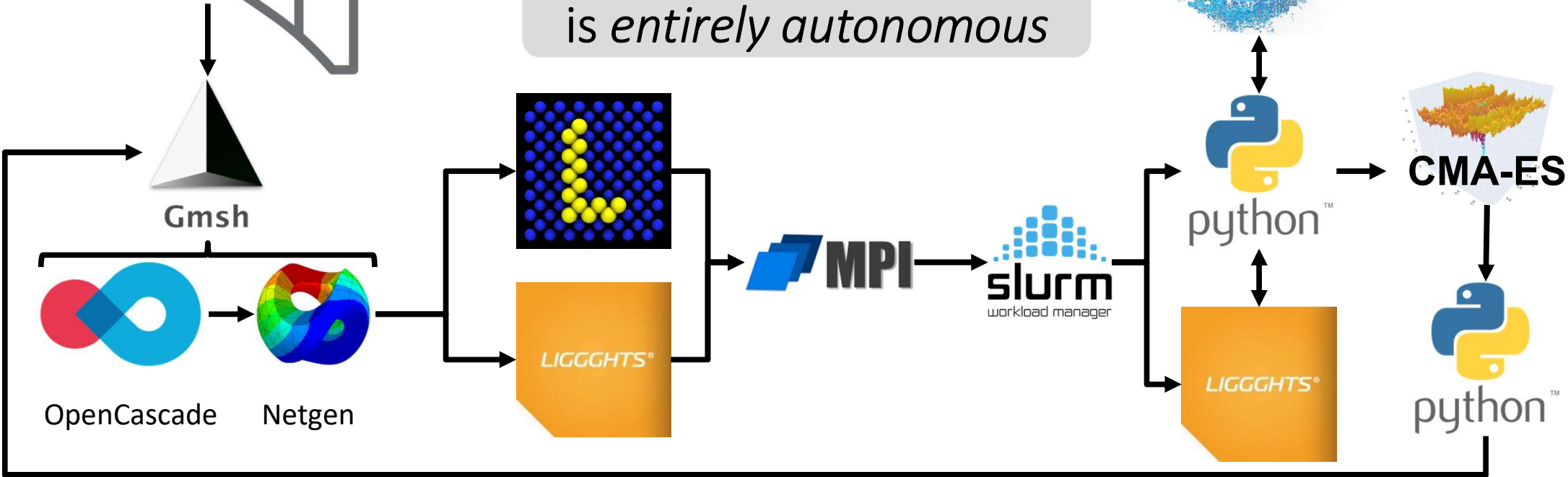
# Getting multiple technologies to “talk”



# Getting multiple technologies to “talk”



Every step in this process is *entirely autonomous*

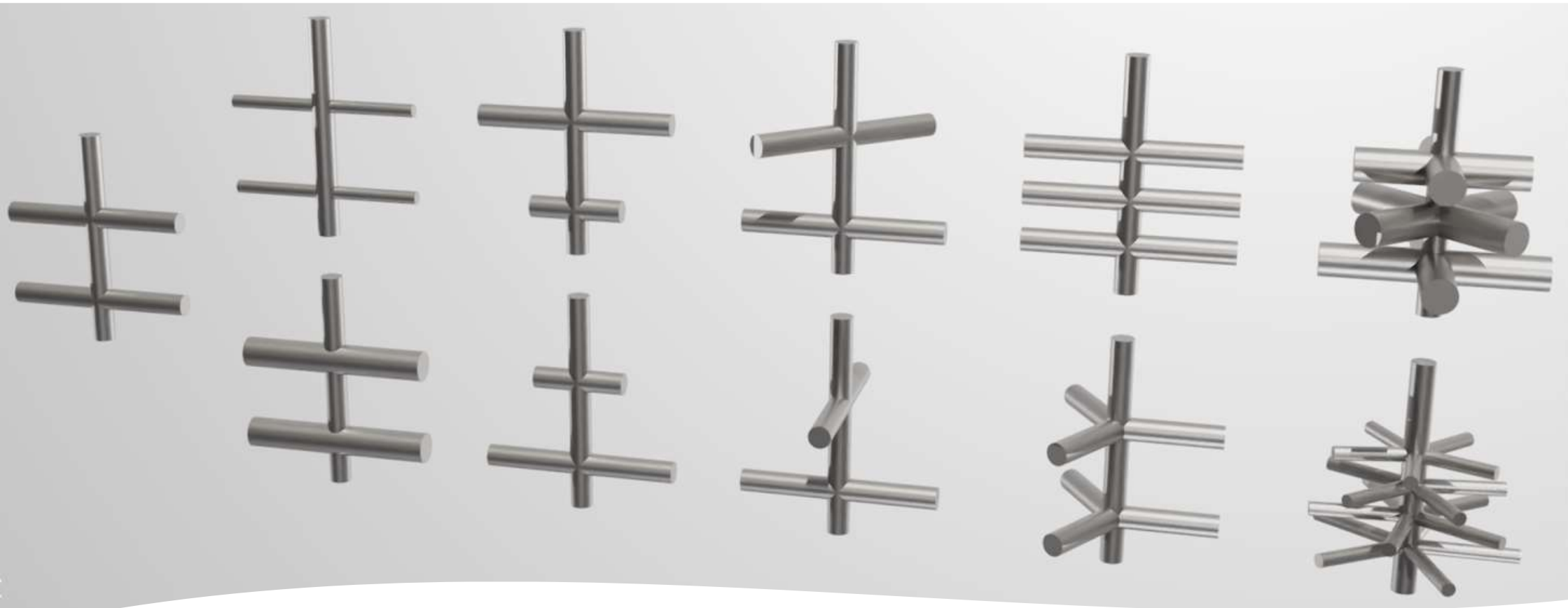


Designing geometry

Creating DEM simulation of geometry

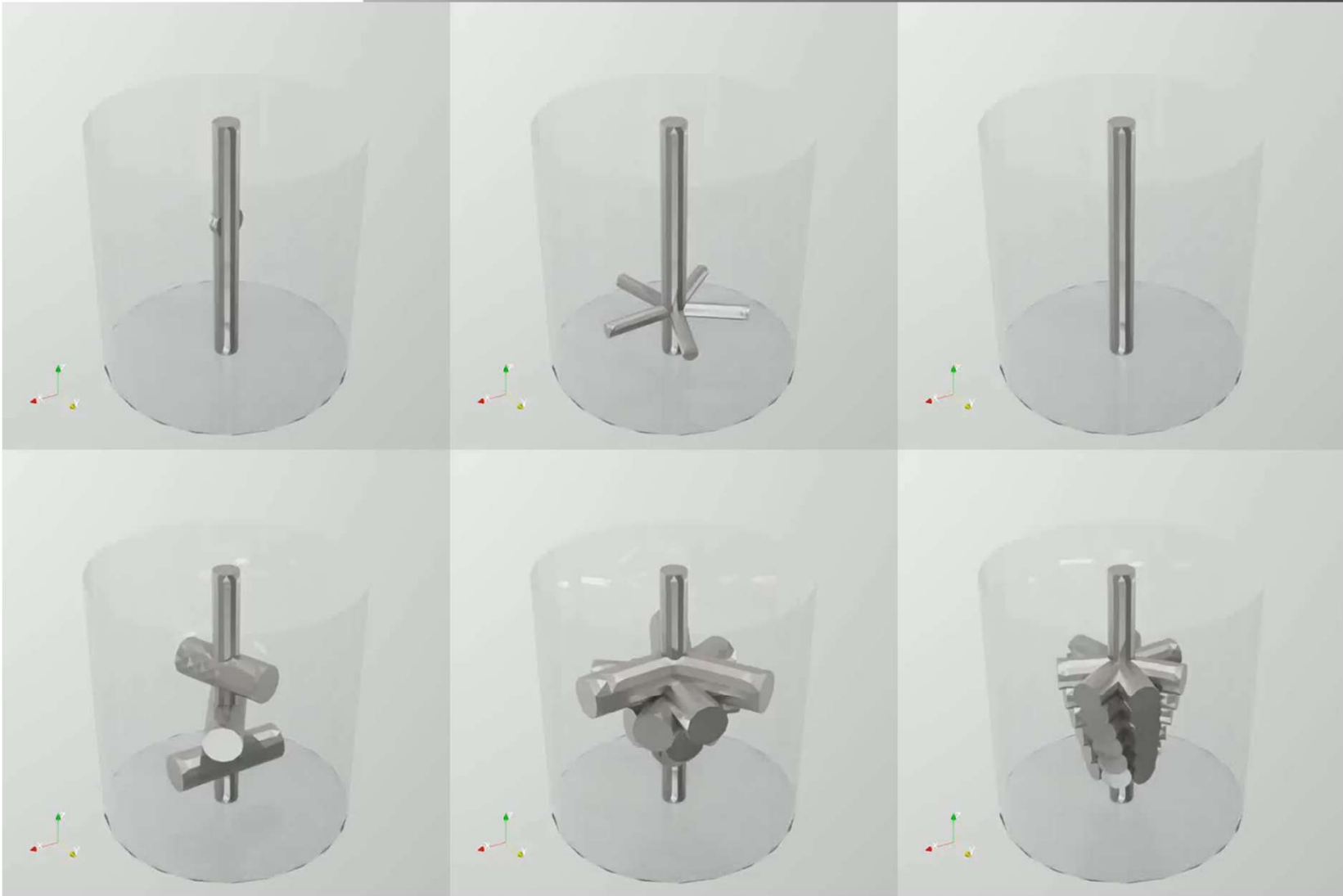
Parallelising

Analysing & optimising



## Case Study

- Optimising a simple attritor mill
- Give HARPPP the ability to vary **pin length, pin diameter, pin number (horizontal and vertical), and pin angle**
- Set goal to minimise power draw → **reduced energy costs, “greener”, more sustainable process**



# What went wrong?

- Technically, **nothing**
- HARPPP did exactly what we requested and **perfectly minimised power draw**
- Nonetheless, it is decidedly *not* a good mill!
- Take home point: need to thoughtfully define our objective

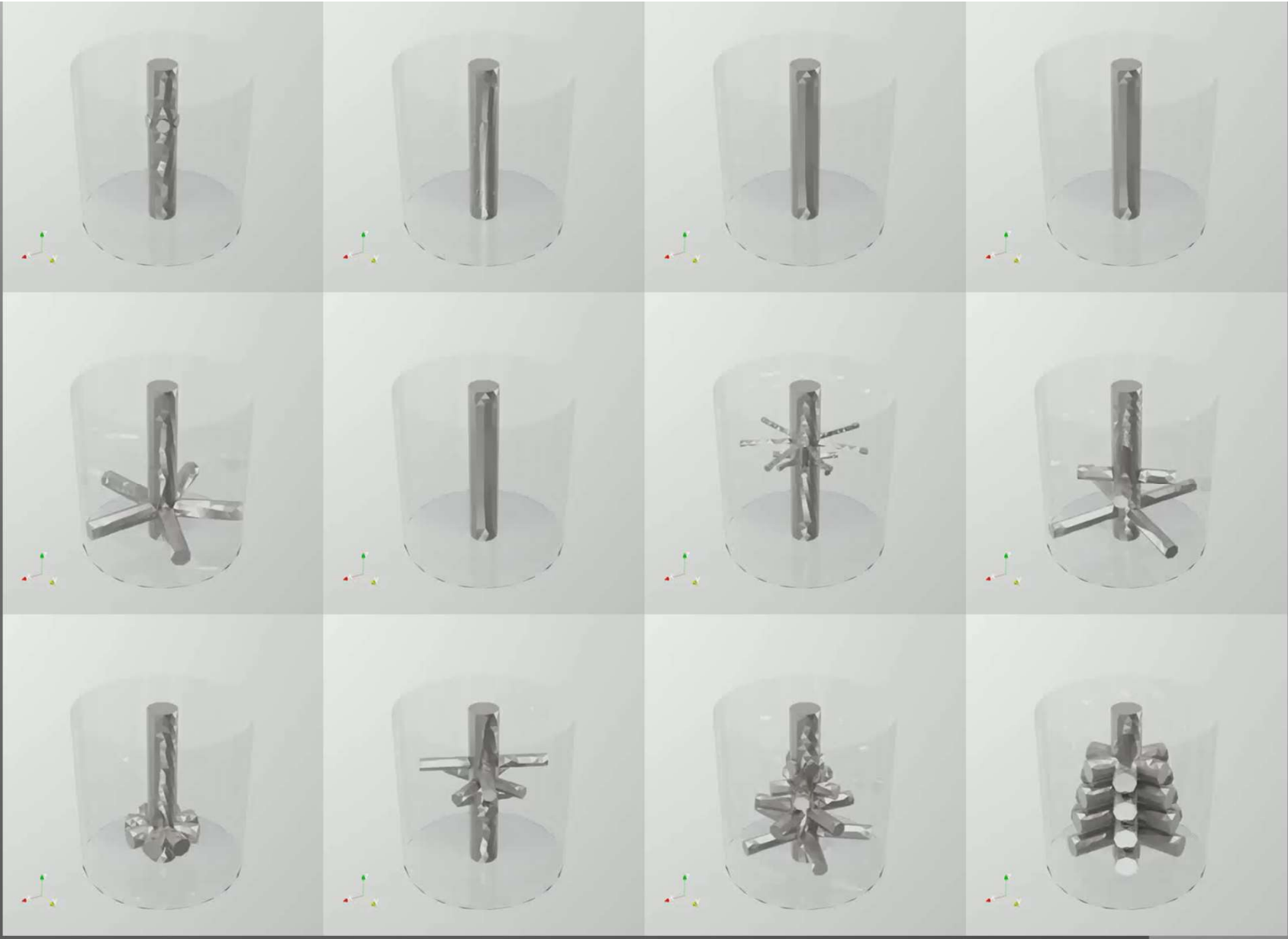




# What went wrong?

- Luckily, HARPPP is capable of **multi-objective optimization**
- Can thus define a more intelligent goal, for example minimize power draw (Objective 1) whilst maintaining a minimum mean pair stress energy (Objective 2)



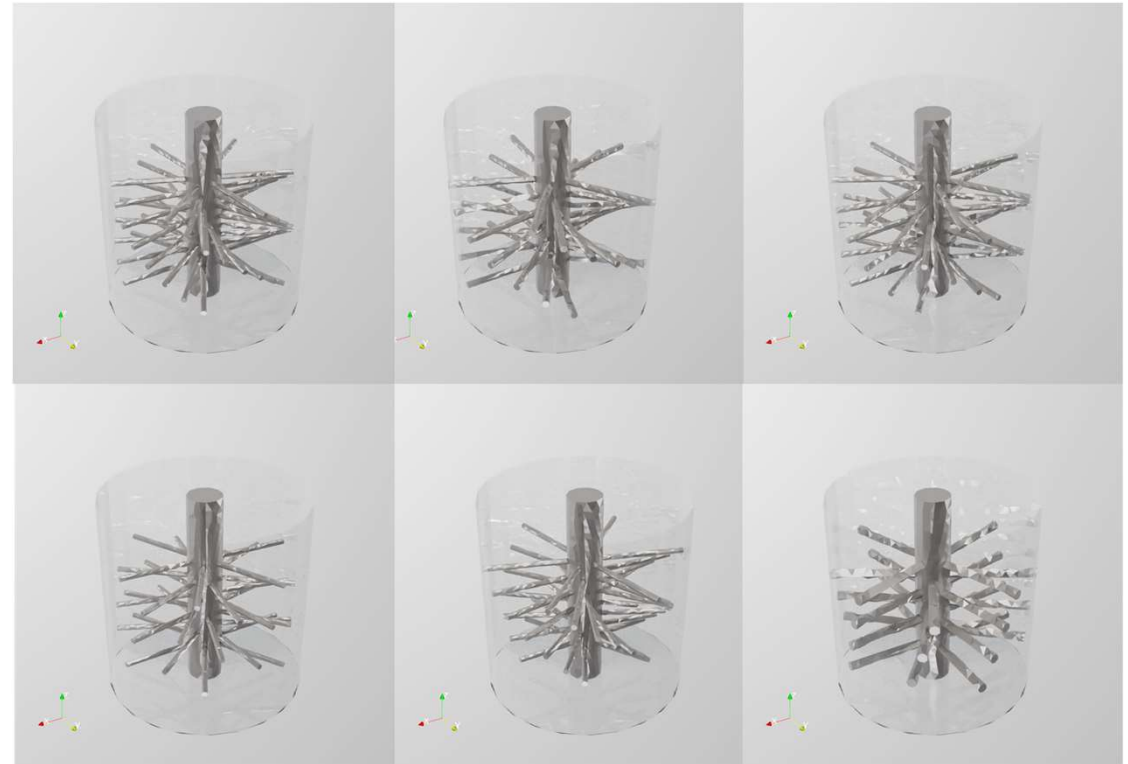


# Results

For different mill geometries, operating conditions and particle properties tested, **energy savings of between 24% and 40%** achieved compared to base model, whilst producing the **same or greater** average pair stress energy

# Can we learn from the machines?

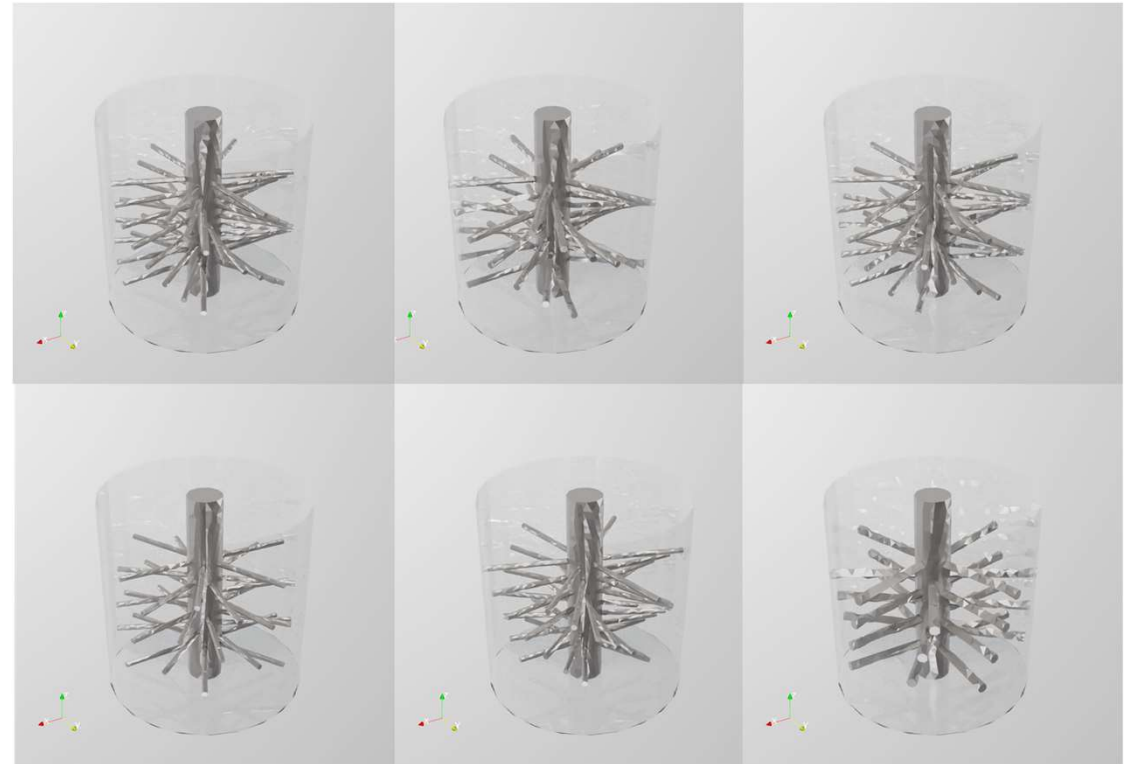
- In later stages of evolution, majority of attritors show certain commonalities, namely **large numbers of long, thin, staggered pins**
- Indeed, these features remain robust even with mills and particles at different scales!
- Does this suggest some key design principles that we can learn from HARPPP?

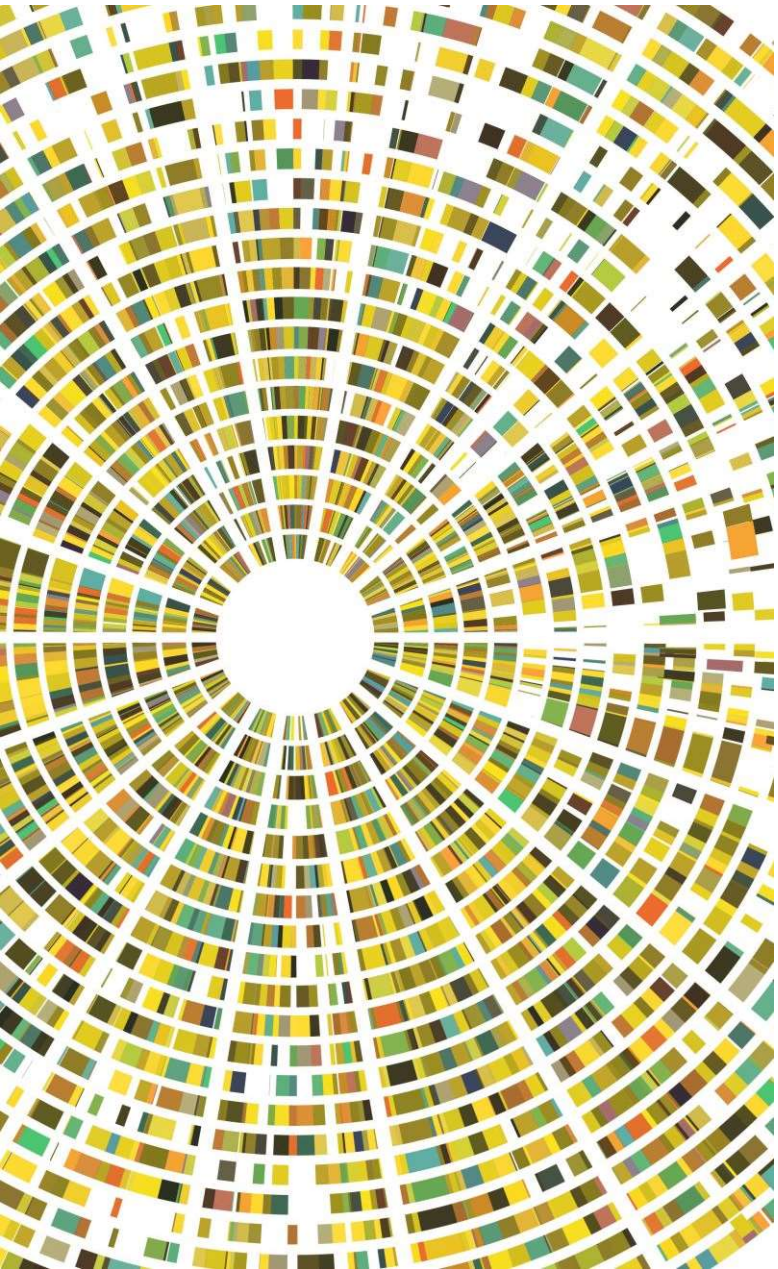


# Can we learn from the machines?

Possible interpretations:

1. Long pins  $\rightarrow$  value fairly obvious!
2. Thin pins  $\rightarrow$  minimise propensity to simply “push” particles  $\rightarrow$  remove interactions which cause power draw without inducing collision or shear
3. Large numbers of closely-packed, staggered pins  $\rightarrow$  redirect particle motion  $\rightarrow$  improve axial transport, induce “chaos”



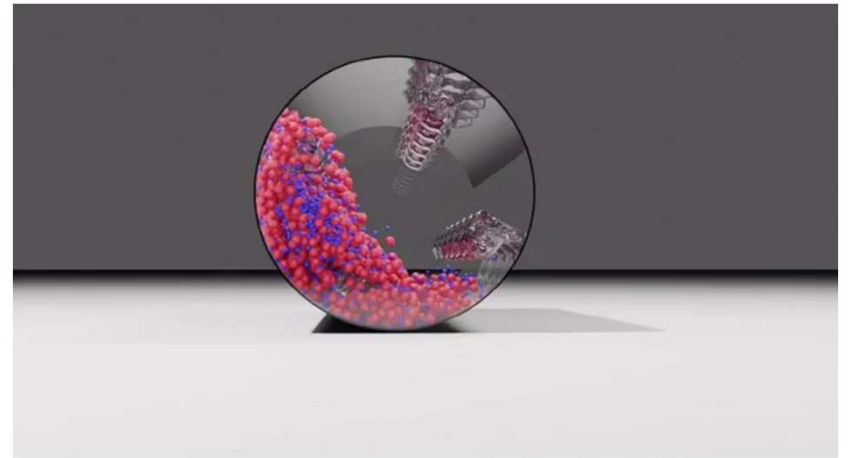


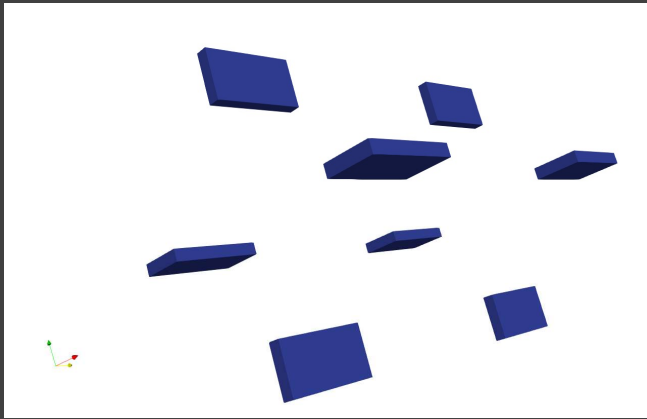
## Only the tip of the iceberg

- A good proof of concept, but still relatively simple.
- Many additional factors which can be included:
  - Lower-bound on pin size to ensure robustness?
  - More complex goals – specific force *distribution*?  
Optimise both collision and shear?
  - More complex geometric variations?
  - ...
- User can define arbitrary number of objectives & constraints dependent on their system, and their goals.
- HARPPP can also, of course, optimise much more than just mills!

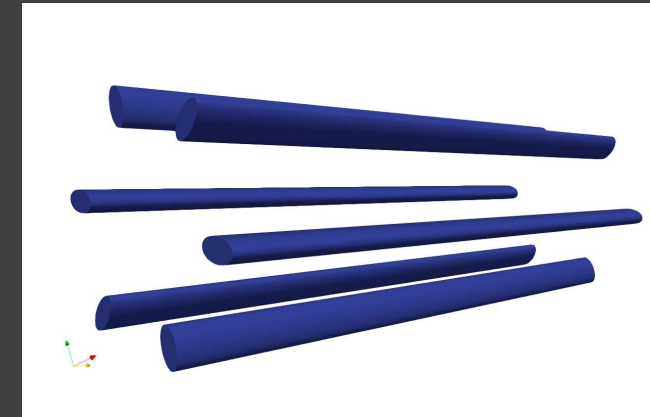
# Not just mills...

- Rotating drum system with particles of differing size & density
- In standard form, significant segregation
- **Goal: design baffles to optimize mixing**



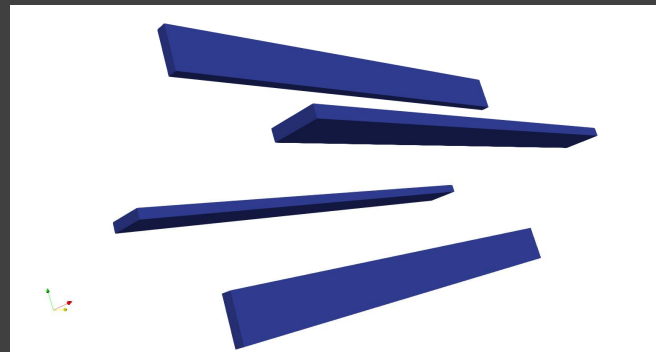
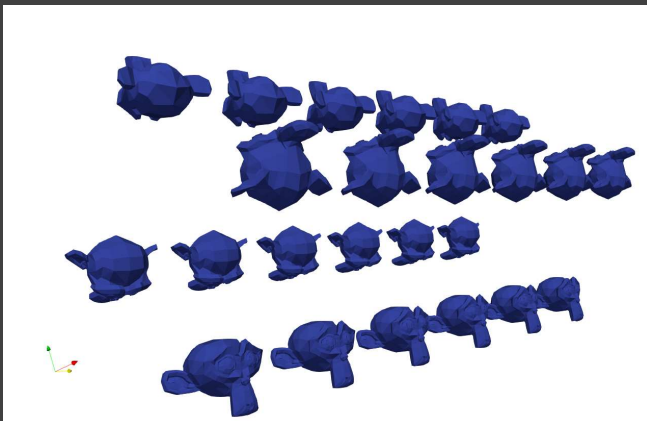


Optimise width, thickness,  
number and axial position



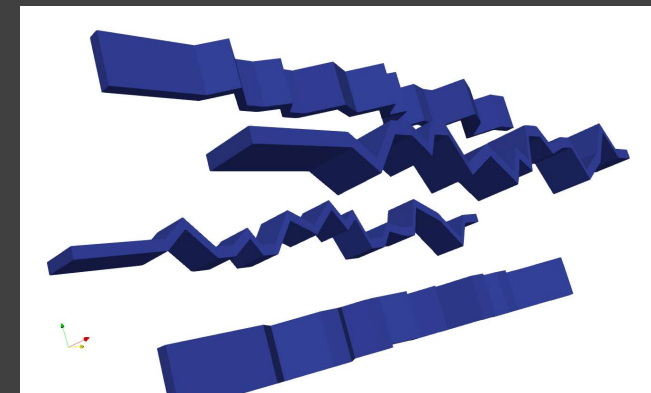
Optimise width, thickness,  
number radial position & shape

Optimise size, number & position  
constrain shape to monkey



Optimise width and thickness,  
constrain number and shape

Optimise width, thickness, &  
local angle



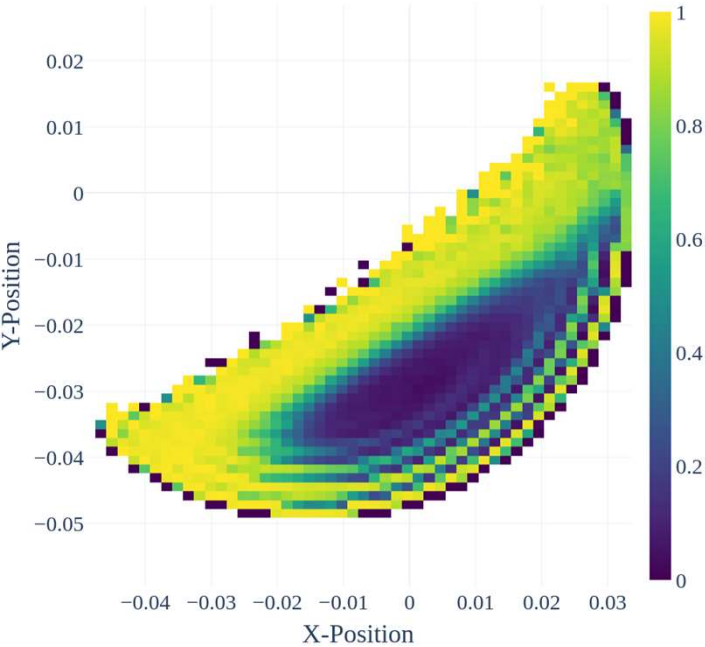


# Optimising Mixing

- Significant segregation in base model (Case 1)
- Marked improvement in Case 2, but geometric constraint and/or radial constraint prevents full optimization
- Case 3 achieves near-perfect mixing

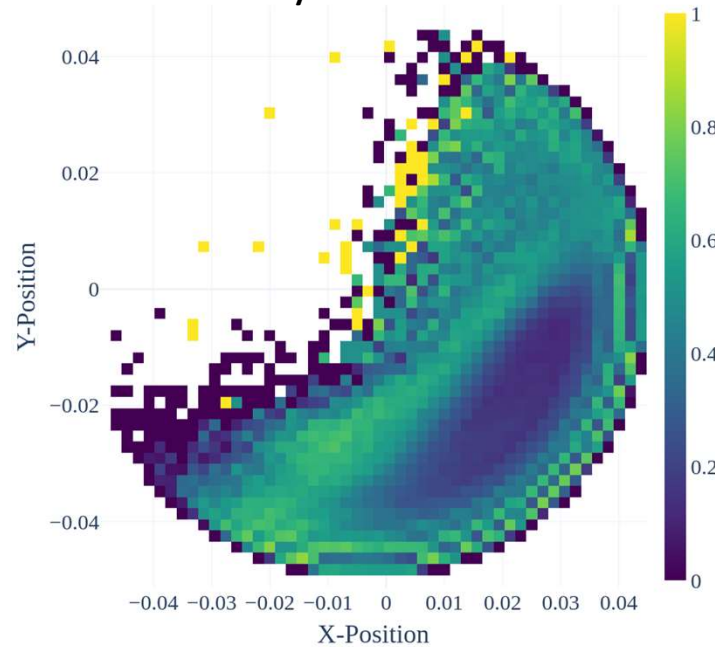
Case 1

Base Model:  $M = 0.80$



Case 2

Monkeys:  $M = 0.96$



Case 3

Rods:  $M = 0.99$

